Gauged (super)-AdS-Maxwell algebra and (super)-gravity

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Belgrad 2012

MacDowell–Mansouri construction

Action:

$$S = \int B^{IJ} \wedge F_{IJ} - \frac{\beta}{2} B^{IJ} \wedge B_{IJ} - \frac{\alpha}{4} \epsilon_{IJKL} B^{IJ} \wedge B^{KL} \tag{1}$$

The algebra has a form:

$$[\mathcal{P}_a, \mathcal{P}_b] = -i\eta_{44}\mathcal{M}_{ab}\,, \qquad [\mathcal{M}_{ab}, \mathcal{P}_c] = -i(\eta_{ac}\mathcal{P}_b - \eta_{bc}\mathcal{P}_a)\,, \tag{2}$$

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = -i(\eta_{ac}\mathcal{M}_{bd} + \eta_{bd}\mathcal{M}_{ac} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad}). \tag{3}$$

The gauged reductive Cartan's connection can be written as:

$$A_\mu^{IJ} = \frac{1}{2}\omega_\mu^{ab}\mathcal{M}_{ab} + \frac{1}{\ell} e_\mu^a \mathcal{P}_a \tag{4}$$

[L. Freidel, A. Starodubtsev, D. Wise]

The BF reformulation of the MacDowell–Mansouri theory

It is possible to reformulate MacDowell–Mansouri action in the BF theory framework.

$$S = \int B^{IJ} \wedge F_{IJ} - \frac{\beta}{2} B^{IJ} \wedge B_{IJ} - \frac{\alpha}{4} \epsilon_{IJKL} B^{IJ} \wedge B^{KL} \qquad (5)$$

Note that one can choose $\alpha \neq \beta$ (The choice $\alpha = \beta$ will lead to a self dual formulation of gravity).
The physical meaning of constants is:

$$\frac{1}{l^2} = \frac{\Lambda}{3} \qquad \alpha = \frac{G\Lambda}{3(1-\gamma^2)} \qquad \beta = \frac{G\Lambda\gamma}{3(1-\gamma^2)} \qquad (6)$$

$$G\Lambda \sim 10^{-120} \qquad (7)$$

[L. Freidel, A. Starodubtsev]

MacDowell–Mansouri construction

$$S = S_{H+\Lambda} + \int \left(\frac{\alpha}{4(\alpha^2 - \beta^2)} R^{ij} \wedge R^{kl} \epsilon_{ijkl} - \frac{\beta}{2(\alpha^2 - \beta^2)} R^{ij} \wedge R_{ij} + \frac{1}{\beta} C \right) \tag{8}$$

Additions to the action are the Euler, Pontryagin and Nieh-Yan classes.

$$S_{H+\Lambda} = -\frac{1}{G} \epsilon_{abcd} (R^{ab} \wedge e^c \wedge e^d - \frac{\Lambda}{3} e^a \wedge e^b \wedge e^c \wedge e^d) \tag{9}$$

$$-\frac{2}{G\gamma} R^{ab} \wedge e_a \wedge e_b \tag{10}$$

Supersymmetry $N = 1$

The action is presented as:

$$S = \int tr(\mathbb{B} \wedge \mathbb{F} - \frac{\beta}{2}\mathbb{B} \wedge \mathbb{B} - \frac{\alpha}{2}\mathbb{B} \wedge \gamma^5 \mathbb{B}) \tag{11}$$

where $\mathbb{B} = (B, \mathcal{B})$. The gauge algebra is $OSp(4|1)$.

$$[\mathcal{M}_{ab}, Q_\alpha] = -\frac{i}{2}\,(\gamma_{ab}\,Q)_\alpha,$$
$$[\mathcal{P}_a, Q_\alpha] = -\frac{i}{2}\,\gamma_a\,(Q_\alpha),$$
$$\{Q_\alpha, Q_\beta\} = -\frac{i}{2}(\gamma^{ab})_{\alpha\beta}\,\mathcal{M}_{ab} + i(\gamma^a)_{\alpha\beta}\,\mathcal{P}_a,$$

Supersymmetry $N = 1$

The action is presented as:

$$S = \int tr(\mathbb{B} \wedge \mathbb{F} - \frac{\beta}{2}\mathbb{B} \wedge \mathbb{B} - \frac{\alpha}{2}\mathbb{B} \wedge \gamma^5\mathbb{B}) \quad (12)$$

where $\mathbb{B} = (B, \mathcal{B})$. The gauge algebra is $OSp(4|1)$.
The connection is constructed as:

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu{}^{ij} M_{ij} + \frac{1}{l} e_\mu{}^i P_i + \kappa\bar{\psi}_\mu Q \quad (13)$$

The curvature has a bosonic and fermionic part:

$$\mathbb{F}_{\mu\nu} = \frac{1}{2}\, F_{\mu\nu}^{(s)\,IJ}\, M_{IJ} + \bar{\mathcal{F}}_{\mu\nu} Q\,. \quad (14)$$

[R. Durka, J. Kowalski-Glikman, M.Sz.]

Supersymmetry $N = 1$

Final Lagrangian contains of SUGRA part with Cosmological Constants (Λ) and the supersymmetric Holst term.

$$L = \frac{1}{G}\,\epsilon^{\mu\nu\rho\sigma}\epsilon_{ijkl}\left(R_{\mu\nu}{}^{ij}\,e_\rho{}^k\,e_\sigma{}^l - \frac{\Lambda}{3}\,e_\mu{}^i\,e_\nu{}^j\,e_\rho{}^k\,e_\sigma{}^l\right) \tag{15}$$

$$-\,\epsilon^{\mu\nu\rho\sigma}\left(\frac{1}{2}\,\bar{\psi}_\mu\,\gamma_5\,\gamma_i\,e^i_\nu D^\omega_\rho\psi_\sigma \;-\frac{i}{4\ell}\,\bar{\psi}_\mu\,\gamma_5\,\gamma_{ij}\,e^i_\nu e^j_\rho\,\psi_\sigma\right) \tag{16}$$

$$+\,\epsilon^{\mu\nu\rho\sigma}\left(\frac{2}{\gamma G}\,R_{\mu\nu}{}^{ij}\,e_{\rho i}\,e_{\sigma j} - \frac{i}{4\gamma}\,\bar{\psi}_\mu\,\gamma_i\,\psi_\nu\,D^\omega_\rho e_\sigma{}^i\right). \tag{17}$$

AdS(dS)-Maxwell gravity

The algebra of the $\mathfrak{so}(3,2) \oplus \mathfrak{so}(3,1)$:

$$[\mathcal{P}_a, \mathcal{P}_b] = -i\eta_{44}\mathcal{M}_{ab} + ik\mathcal{Z}_{ab} = i(\lambda^2\mathcal{M}_{ab} - n\mathcal{Z}_{ab})\,, \qquad (18)$$

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = -i(\eta_{ac}\mathcal{M}_{bd} + \eta_{bd}\mathcal{M}_{ac} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad}). \qquad (19)$$

$$[\mathcal{M}_{ab}, \mathcal{Z}_{cd}] = -i(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}), \qquad (20)$$

$$[\mathcal{Z}_{ab}, \mathcal{Z}_{cd}] = -in^{-1}(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}), \qquad (21)$$

$$[\mathcal{M}_{ab}, \mathcal{P}_c] = -i(\eta_{ac}\mathcal{P}_b - \eta_{bc}\mathcal{P}_a), \qquad [\mathcal{Z}_{ab}, \mathcal{P}_c] = 0\,. \qquad (22)$$

Change of cosmological const. λ^2

$$\mathcal{AdS} \xrightarrow{\lambda^2 \to -\lambda^2} d\mathcal{S} \qquad (23)$$

$$\mathfrak{so}(3,2) \oplus \mathfrak{so}(3,1) \xrightarrow{\lambda^2 \to -\lambda^2} \mathfrak{so}(4,1) \oplus \mathfrak{so}(3,1) \qquad (24)$$

[R. Durka, J. Kowalski-Glikman, M.Sz.]

AdS(dS)-Maxwell gravity

If one use isomorphism $\mathcal{Z} \leftarrow \mathcal{M} - \mathcal{Z}$ then algebra become[1]:

$$[\mathcal{P}_a, \mathcal{P}_b] = n\mathcal{Z}_{ab}\,, \qquad (25)$$

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = -i(\eta_{ac}\mathcal{M}_{bd} + \eta_{bd}\mathcal{M}_{ac} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad}). \qquad (26)$$

$$[\mathcal{M}_{ab}, \mathcal{Z}_{cd}] = -i(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}), \qquad (27)$$

$$[\mathcal{Z}_{ab}, \mathcal{Z}_{cd}] = -in^{-1}\lambda^2(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}), \qquad (28)$$

$$[\mathcal{M}_{ab}, \mathcal{P}_c] = -i(\eta_{ac}\mathcal{P}_b - \eta_{bc}\mathcal{P}_a), \qquad [\mathcal{Z}_{ab}, \mathcal{P}_c] = -in^{-1}\lambda^2(\eta_{ac}\mathcal{P}_b - \eta_{bc}\mathcal{P}_a)\,. \qquad (29)$$

[D. Soroka, V. Soroka]

Change of cosmological const. λ^2

$$\mathcal{A}d\mathcal{S} \xrightarrow{\lambda^2 \to -\lambda^2} d\mathcal{S} \qquad (30)$$

[1] [R. Durka, J. Kowalski-Glikman, M.Sz.]

AdS-Maxwell gravity

Let me take a Wigner–Inonu contraction in the form

$$\mathcal{Z} \to \lambda^2 \mathcal{Z} \qquad \mathcal{P} \to \lambda \mathcal{P} \tag{31}$$

then algebra becomes

$$[\mathcal{P}_a, \mathcal{P}_b] = i\mathcal{Z}_{ab} \qquad [\mathcal{Z}_{ab}, \mathcal{Z}_{cd}] = 0, \ , \tag{32}$$

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = -i(\eta_{ac}\mathcal{M}_{bd} + \eta_{bd}\mathcal{M}_{ac} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad}), \tag{33}$$

$$[\mathcal{M}_{ab}, \mathcal{Z}_{cd}] = -i(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}), \tag{34}$$

$$[\mathcal{M}_{ab}, \mathcal{P}_c] = -i(\eta_{ac}\mathcal{P}_b - \eta_{bc}\mathcal{P}_a), \qquad [\mathcal{Z}_{ab}, \mathcal{P}_c] = 0\,. \tag{35}$$

a Maxwell algebra.

[R. Durka, J. Kowalski-Glikman, M.Sz.]

AdS-Maxwell gravity

The connection

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu{}^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}e^a_\mu\mathcal{P}_a + \frac{1}{2}h^{ab}_\mu\mathcal{Z}_{ab} \tag{36}$$

corresponds to curvature

$$\mathbb{F}_{\mu\nu} = \frac{1}{2}\,F^{ab}_{\mu\nu}\,\mathcal{M}_{ab} + \frac{1}{\ell}T^a_{\mu\nu}\,\mathcal{P}_a + \frac{1}{2}\,G^{ab}_{\mu\nu}\,\mathcal{Z}_{ab}\,. \tag{37}$$

The art of the Action

$$\begin{aligned} 16\pi\, S(A,B) \quad = \quad & \int 2(B^{a4}\wedge F_{a4} - \frac{\beta}{2}B^{a4}\wedge B_{a4}) \\ & + B^{ab}\wedge F_{ab} - \frac{\beta}{2}B^{ab}\wedge B_{ab} - \frac{\alpha}{4}\epsilon^{abcd}B_{ab}\wedge B_{cd} \\ & + C^{ab}\wedge G_{ab} - \frac{\rho}{2}C^{ab}\wedge C_{ab} - \frac{\sigma}{4}\epsilon^{abcd}C_{ab}\wedge C_{cd} \\ & - \beta C^{ab}\wedge B_{ab} - \frac{\alpha}{2}\epsilon^{abcd}C_{ab}\wedge B_{cd} \end{aligned} \tag{38}$$

AdS-Maxwell gravity

The art of the Action

$$\begin{aligned} 16\pi\, S(A,B) &= \int 2(B^{a4}\wedge F_{a4} - \frac{\beta}{2} B^{a4}\wedge B_{a4}) \\ &+B^{ab}\wedge F_{ab} - \frac{\beta}{2} B^{ab}\wedge B_{ab} - \frac{\alpha}{4}\epsilon^{abcd} B_{ab}\wedge B_{cd} \\ &+C^{ab}\wedge G_{ab} - \frac{\rho}{2} C^{ab}\wedge C_{ab} - \frac{\sigma}{4}\epsilon^{abcd} C_{ab}\wedge C_{cd} \\ &-\beta C^{ab}\wedge B_{ab} - \frac{\alpha}{2}\epsilon^{abcd} C_{ab}\wedge B_{cd} \qquad (39) \end{aligned}$$

becomes the gravity plus generalized topological term:

$$\begin{aligned} 16\pi S(\omega,h,e) &= \int \left(\frac{1}{4} M^{abcd} F_{ab}\wedge F_{cd} - \frac{1}{\beta\ell^2} T^a \wedge T_a\right) \qquad (40) \\ &+ \int \frac{1}{4} N^{abcd} (G_{ab}+F_{ab})\wedge(G_{ab}+F_{ab}) \end{aligned}$$

[R. Durka, J. Kowalski-Glikman, M.Sz.]

AdS-self-Yang-Mills theory

Let consider base transformation $\tilde{\mathcal{M}} \leftarrow \mathcal{M} - \mathcal{Z}$, which corespont to field transformation $\varpi \leftarrow \omega + h$, then the connection become

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu{}^{ab}\tilde{\mathcal{M}}_{ab} + \frac{1}{\ell}e^a_\mu \mathcal{P}_a - \frac{1}{2}\varpi^{ab}_\mu \mathcal{Z}_{ab} \ . \tag{41}$$

Algebra is separated clearly

$$\Big(\mathfrak{so}(3,2) \equiv AdS[\mathcal{M},\mathcal{P}]\Big) \oplus \Big(\mathcal{A}[\mathcal{M},\mathcal{Z}] \equiv \mathfrak{so}(3,1)\Big) \tag{42}$$

Now one can see that

$$\mathfrak{so}(3,2) \oplus \mathfrak{so}(3,1) \simeq \mathfrak{so}(3,2) \oplus (\mathfrak{su}(2)^+ \times \mathfrak{su}(2)^-) \tag{43}$$

$$\tilde{F}[\varpi] = F[\omega] + G[h] \quad \text{and} \quad \tilde{F}^- = 0 \tag{44}$$

[A. Borowiec, M.Sz]

AdS–(self-dual)–Yang–Mills theory

$$
\begin{aligned}
S &= S_E + S_1 + S_2 \\
&= \frac{1}{64\pi G} \int \epsilon_{abcd} \left(R_{\mu\nu}{}^{ab} e^c_\rho e^d_\sigma - \frac{\Lambda}{3} e^a_\mu e^b_\nu e^c_\rho e^d_\sigma \right) \epsilon^{\mu\nu\rho\sigma} \\
&- \frac{1}{4} \int e \, \left(\varrho_1 \, \tilde{F}_{\mu\nu}{}^{ab} \tilde{F}^{\mu\nu}{}_{ab} + \varrho_2 \, \tilde{F}_{\mu\nu}{}^{ab} \tilde{F}^{\mu\nu cd} \, \epsilon_{abcd} \right) . \qquad (45)
\end{aligned}
$$

The field equations resulting from this action are Einstein eq.

$$
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda \, g_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad (46)
$$

$$
\begin{aligned}
T_{\mu\nu} = \varrho_1 &\left(\tilde{F}_{\mu\lambda}{}^{ab} \tilde{F}_\nu{}^\lambda{}_{ab} - \frac{1}{4} g_{\mu\nu} \, \tilde{F}_{\lambda\sigma}{}^{ab} \tilde{F}^{\lambda\sigma}{}_{ab} \right) \\
+ \varrho_2 &\left(\tilde{F}_{\mu\lambda}{}^{ab} \tilde{F}_\nu{}^{\lambda cd} - \frac{1}{4} g_{\mu\nu} \, \tilde{F}_{\lambda\sigma}{}^{ab} \tilde{F}^{\lambda\sigma cd} \right) \epsilon_{abcd} \qquad (47)
\end{aligned}
$$

[R. Durka, J. Kowalski-Glikman, A. Borowiec, M.Sz]

Super-AdS-Maxwell gravity

The super algebra has a form

$$[\mathcal{M}_{ab}, Q_\alpha] = -\frac{i}{2}\,(\gamma_{ab}\,Q)_\alpha\,, [\mathcal{M}_{ab}, \Sigma_\alpha] = -\frac{i}{2}\,(\gamma_{ab}\,\Sigma)_\alpha\,,$$

$$[\mathcal{P}_a, Q_\alpha] = -\frac{i}{2}\,\gamma_a\,(Q_\alpha - \Sigma_\alpha)\,, \qquad [\mathcal{P}_a, \Sigma_\alpha] = 0,$$

$$\{Q_\alpha, Q_\beta\} = -\frac{i}{2}(\gamma^{ab})_{\alpha\beta}\,\mathcal{M}_{ab} + i(\gamma^a)_{\alpha\beta}\,\mathcal{P}_a\,,$$

$$\{Q_\alpha, \Sigma_\beta\} = -\frac{i}{2}(\gamma^{ab})_{\alpha\beta}\,\mathcal{Z}_{ab}\,,\ \{\Sigma_\alpha, \Sigma_\beta\} = -\frac{i}{2}(\gamma^{ab})_{\alpha\beta}\,\mathcal{Z}_{ab}$$

$$[\mathcal{Z}_{ab}, Q_\alpha] = -\frac{i}{2}\,(\gamma_{ab}\,\Sigma)_\alpha\,, \qquad [\mathcal{Z}_{ab}, \Sigma_\alpha] = -\frac{i}{2}\,(\gamma_{ab}\,\Sigma)_\alpha \qquad .$$

[R. Durka, J. Kowalski-Glikman, M.Sz.]

Super-AdS-Maxwell gravity

The gauge field is

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}e_\mu^a\mathcal{P}_a + \frac{1}{2}h_\mu^{ab}\mathcal{Z}_{ab} + \kappa\bar{\psi}_\mu^\alpha Q_\alpha + \tilde{\kappa}\bar{\chi}_\mu^\alpha\Sigma_\alpha \tag{48}$$

and the most general "rotation" invariant action is

$$\begin{aligned}
64\pi\mathcal{L} = & \left(B_{\mu\nu}^{IJ}F^{(s)}_{\rho\sigma\,IJ} - \frac{\beta}{2}\,B_{\mu\nu}^{IJ}B_{\rho\sigma\,IJ} - \frac{\alpha}{4}\epsilon_{abcd}B_{\mu\nu}^{ab}B_{\rho\sigma}^{cd}\right)\epsilon^{\mu\nu\rho\sigma} \\
& + \left(C_{\mu\nu}^{ab}G^{(s)}_{\rho\sigma\,ab} - \frac{\rho}{2}\,C_{\mu\nu}^{ab}C_{\rho\sigma\,ab} - \frac{\sigma}{4}\epsilon_{abcd}C_{\mu\nu}^{ab}C_{\rho\sigma}^{cd}\right)\epsilon^{\mu\nu\rho\sigma} \\
& + \left(\beta\,C_{\mu\nu}^{ab}B_{\rho\sigma\,ab} + \frac{\alpha}{2}\epsilon_{abcd}C_{\mu\nu}^{ab}B_{\rho\sigma}^{cd}\right)\epsilon^{\mu\nu\rho\sigma} \\
& +4\left(\bar{\mathcal{B}}_{\mu\nu}\mathcal{F}_{\rho\sigma} - \frac{\beta}{2}\,\bar{\mathcal{B}}_{\mu\nu}\mathcal{B}_{\rho\sigma} - \frac{\alpha}{2}\,\bar{\mathcal{B}}_{\mu\nu}\gamma^5\mathcal{B}_{\rho\sigma}\right)\epsilon^{\mu\nu\rho\sigma} \\
& +4\left(\bar{\mathcal{C}}_{\mu\nu}\mathcal{G}_{\rho\sigma} - \frac{\rho}{2}\,\bar{\mathcal{C}}_{\mu\nu}\mathcal{C}_{\rho\sigma} - \frac{\sigma}{2}\,\bar{\mathcal{C}}_{\mu\nu}\gamma^5\mathcal{C}_{\rho\sigma}\right)\epsilon^{\mu\nu\rho\sigma} \\
& +4\left(\frac{\beta}{2}\,\bar{\mathcal{C}}_{\mu\nu}\mathcal{B}_{\rho\sigma} + \frac{\beta}{2}\,\bar{\mathcal{B}}_{\mu\nu}\mathcal{C}_{\rho\sigma} + \frac{\alpha}{2}\,\bar{\mathcal{C}}_{\mu\nu}\gamma^5\mathcal{B}_{\rho\sigma} + \frac{\alpha}{2}\,\bar{\mathcal{B}}_{\mu\nu}\gamma^5\mathcal{C}_{\rho\sigma}\right)\epsilon^{\mu\nu\rho\sigma}\,.
\end{aligned} \tag{49}$$

Super-AdS-Maxwell gravity

The Lagrangian reduces therefore to the final form

$$
\begin{aligned}
16\pi\mathcal{L} = & \left(\frac{1}{16}M_{abcd}\, F^{ab}_{\mu\nu}\, F^{cd}_{\rho\sigma} - \frac{1}{4\beta\ell^2}T^{a}_{\mu\nu}\, T_{\rho\sigma\, a}\right)\, \epsilon^{\mu\nu\rho\sigma} \\
& - \left(\frac{\kappa^2}{G}\,\bar{\psi}_\mu\, \gamma^5\gamma_{ab}\, e^a_\nu\, e^b_\rho + \frac{2\kappa^2\ell}{G}\,\bar{\psi}_\mu\, \gamma^5\gamma_a\, e^a_\nu\, \mathcal{D}^\omega_\rho \psi_\sigma\right)\, \epsilon^{\mu\nu\rho\sigma} \\
& + \frac{\kappa^2\ell}{2\gamma G}\,\bar{\psi}_\mu\gamma_a\, \psi_\nu\, T^a_{\rho\sigma}\, \epsilon^{\mu\nu\rho\sigma} + \textit{total derivative}\,,
\end{aligned}
\qquad (50)
$$

[R. Durka, J. Kowalski-Glikman, M.Sz.]

Extended AdS-Maxwell theory

The algebra $\mathfrak{so}(3,2) \oplus \mathfrak{so}(3,1) \oplus \mathfrak{so}(3,2) \oplus \mathfrak{so}(3,1)$ has a form:

$$[\mathcal{P}_a, \mathcal{P}_b] = -i\eta_{44}\mathcal{M}_{ab} + ik\mathcal{Z}_{ab} = i(\mathcal{M}_{ab} + k\mathcal{Z}_{ab})\,, \qquad (51)$$

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = -i(\eta_{ac}\mathcal{M}_{bd} + \eta_{bd}\mathcal{M}_{ac} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad}). \qquad (52)$$

$$[\mathcal{M}_{ab}, \mathcal{Z}_{cd}] = -i(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}), \qquad (53)$$

$$[\mathcal{Z}_{ab}, \mathcal{Z}_{cd}] = +ik(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}), \qquad (54)$$

$$[\mathcal{M}_{ab}, \mathcal{P}_c] = -i(\eta_{ac}\mathcal{P}_b - \eta_{bc}\mathcal{P}_a), \qquad [\mathcal{Z}_{ab}, \mathcal{P}_c] = 0\,. \qquad (55)$$

$$[\mathcal{R}_a, \mathcal{R}_b] = i\mathcal{Z}_{ab}\,, \qquad (56)$$

$$[\mathcal{M}_{ab}, \mathcal{R}_c] = -i(\eta_{ac}\mathcal{R}_b - \eta_{bc}\mathcal{R}_a) \qquad (57)$$

$$[\mathcal{M}_{ab}, \mathcal{R}_c] = -i(\eta_{ac}\mathcal{R}_b - \eta_{bc}\mathcal{R}_a), \qquad [\mathcal{P}_a, \mathcal{R}_c] = 0\,. \qquad (58)$$

[A.Borowiec, M.Sz.]

Extended AdS-Maxwell theory

The connection

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu{}^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}e^a_\mu\mathcal{P}_a + \frac{1}{2}h^{ab}_\mu\mathcal{Z}_{ab} + \frac{1}{\ell}f^a_\mu\mathcal{R}_a \tag{59}$$

The curvatures has a form

$$F^{ab}_{\mu\nu} = R^{ab}_{\mu\nu} + \frac{1}{\ell^2}(e^a_\mu e^b_\nu - e^a_\nu e^b_\mu)\,, \tag{60}$$

$$T^a_{\mu\nu} = D^\omega_\mu e^a_\nu - D^\omega_\nu e^a_\mu\,, \tag{61}$$

$$\begin{aligned} G^{ab}_{\mu\nu} &= D^\omega_\mu h^{ab}_\nu - D^\omega_\nu h^{ab}_\mu - \frac{1}{\ell^2}(e^a_\mu e^b_\nu - e^a_\nu e^b_\mu) \\ &+ (h^{ac}_\mu h_{\nu\,c}{}^b - h^{ac}_\nu h_{\mu\,c}{}^b) - \frac{1}{\ell^2}(f^a_\mu f^b_\nu - f^a_\nu f^b_\mu)\,, \end{aligned} \tag{62}$$

$$Y^a_{\mu\nu} = D^\omega_\mu f^a_\nu - D^\omega_\nu f^a_\mu + h^{ab}_\mu f_{\nu b} - h^{ab}_\nu f_{\mu b}. \tag{63}$$

and the action is

$$S = S^{(AdS-Maxwell)} + 2\,C^a \wedge Y_a \tag{64}$$

[A. Borowiec, M.Sz.]

Extended AdS-Maxwell theory

The addtional e.o.m. says

$$df^a + \varpi^a{}_b \wedge f^b = \tilde{T}^a = 0 \,. \tag{65}$$

where $\varpi = \omega + h$.

$$S_{E_2} = \frac{1}{64\pi G} \int \epsilon_{abcd} \left(H_{\mu\nu}[\varpi]^{ab} f^c_\rho f^d_\sigma - \frac{\Lambda'}{3} f^a_\mu f^b_\nu f^c_\rho f^d_\sigma \right) \epsilon^{\mu\nu\rho\sigma} \tag{66}$$

[A. Borowiec, M.Sz.]

This is almost $f - g$ model but without

$$S^{int} = M^2(-g)^u(-q)^{(1/2-u)}\, V(g^{-1}q) \tag{67}$$

[R. Durka, J. Kowalski-Glikman]

Thank you for your attention !

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