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Vacuum polarization effects in the background of codimension-2 branes

Yu.A.Sitenko

(BITP, Kyiv, Ukraine)

Outline

1. Spontaneous symmetry breaking and topological defects of $\pi_1 = \mathbb{Z}$

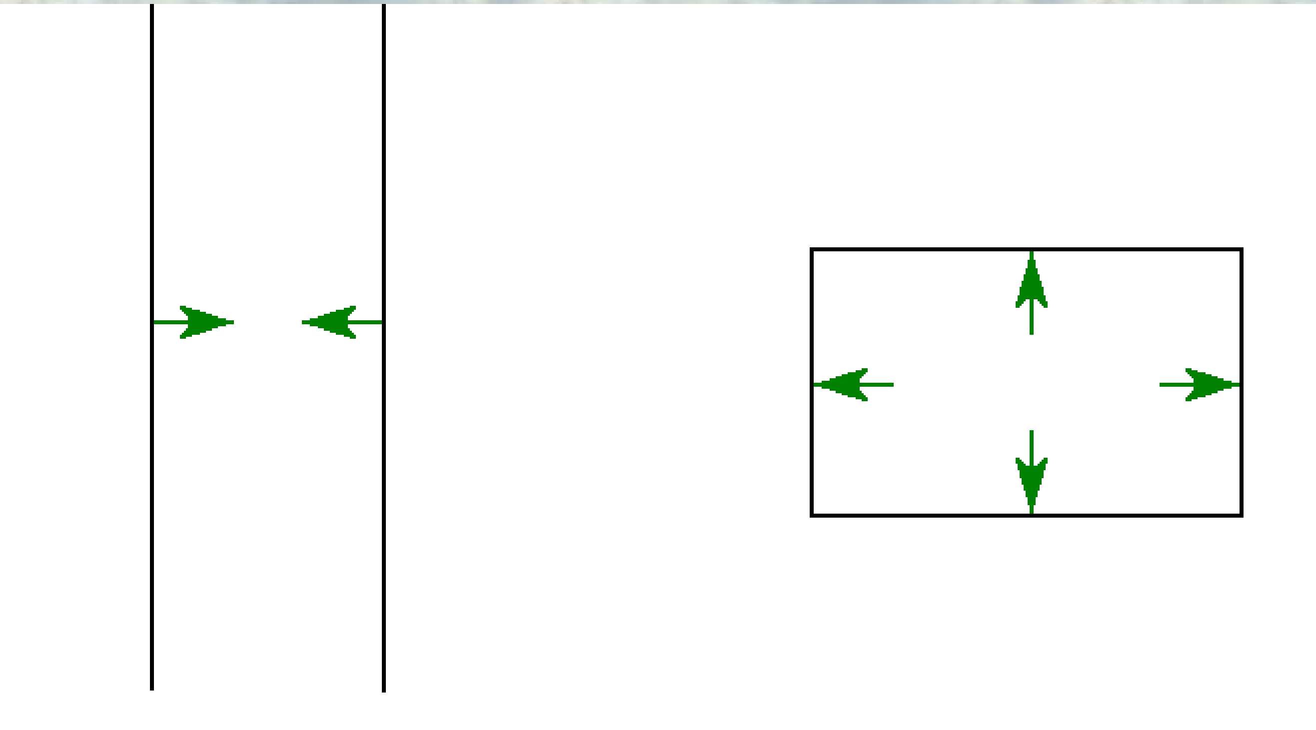
2. Abrikosov-Nielsen-Olesen vortex, cosmic strings and codimension-2 branes

3. Induced vacuum current and magnetic field in the background of codimension-2 branes

4. Induced vacuum energy-momentum tensor in the background of codimension-2 branes

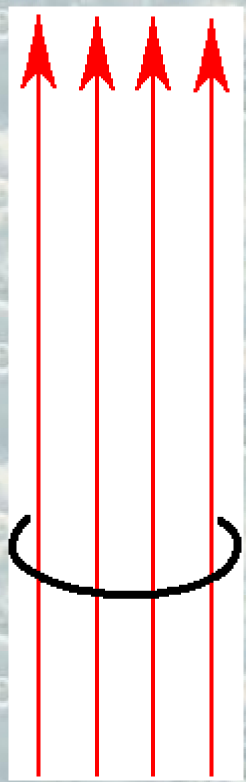
5. Finite transverse size effects

Casimir effect

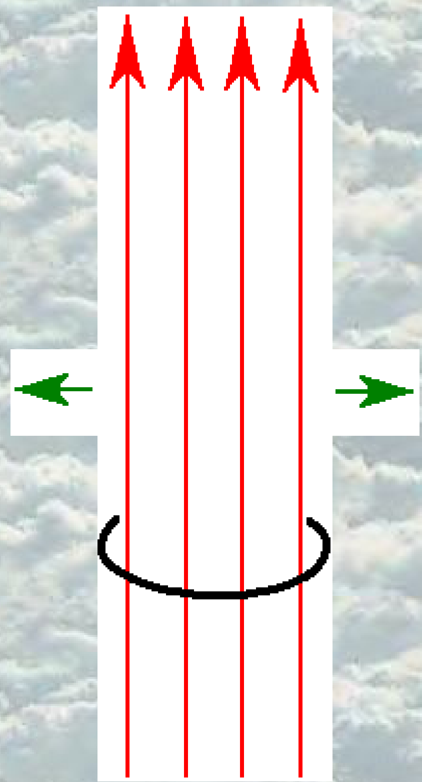


vacuum polarization

Aharonov-Bohm effect

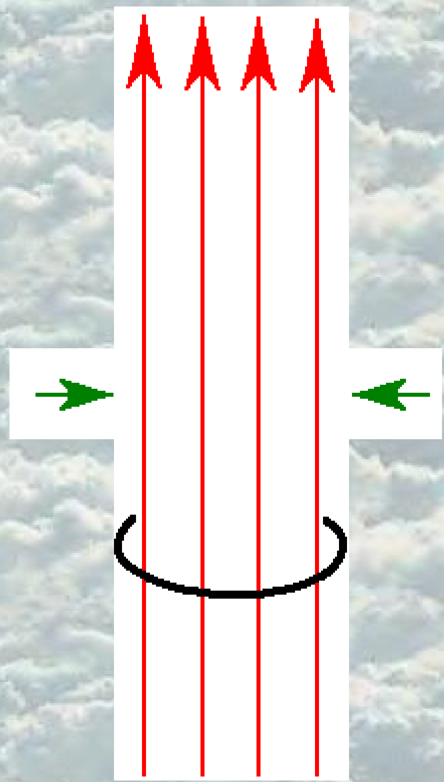


Casimir-Aharonov-Bohm effect



vacuum polarization

Casimir-Aharonov-Bohm effect



vacuum polarization

U(1) gauge Higgs model

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - [(\partial_\mu - ig_H V_\mu)\phi]^*[(\partial^\mu - ig_H V^\mu)\phi] - \lambda(\phi^*\phi - \sigma^2/2)^2$$

$$\phi(x) = \frac{1}{\sqrt{2}}[\sigma + \chi(x)]e^{i\widetilde{\chi}(x)}$$

upon quantization:

$$\langle \mathrm{vac}|\phi|\mathrm{vac}\rangle = \frac{1}{\sqrt{2}}\sigma \mathrm{e}^{\mathrm{i}\vartheta(\mathrm{x})} : \begin{cases} \langle \mathrm{vac}|\chi|\mathrm{vac}\rangle = 0 \\ \langle \mathrm{vac}|\widetilde{\chi}|\mathrm{vac}\rangle = \vartheta(\mathrm{x}) \end{cases}$$

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_V^2 V_\mu V^\mu - \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) - \frac{1}{2}m_V^2\chi^2 + \ldots$$

$$m_V = g_H\,\sigma \qquad m_H = \sqrt{2\lambda}\,\sigma$$

Topological defect of $\pi_1 = \mathbb{Z}$: Abrikosov-Nielsen-Olesen vortex

$$\phi = \frac{1}{\sqrt{2}}\sigma\tau(r)e^{in\varphi}$$

$$\tau(0) = 0 \qquad \tau(\infty) = 1$$

$$V_\varphi = \frac{n}{g_H}\tau^2(r) \qquad (V_r = V_z = V_0 = 0)$$

$$B^3 = \frac{2n}{g_H\, r}\tau\tau'$$

stress-energy tensor

$$T_{00} = -T_{33} = \frac{1}{2g_H^2}\left[(\tau')^2 + \frac{1}{4}(1-\tau^2)^2 m_H^2\right]\left(m_H^2 + 4\frac{\tau^2 n^2}{r^2}\right)$$

$$(m_V = m_H)$$

T_{00} and B_3 are nonvanishing inside the vortex core

Einstein-Hilbert equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

scalar curvature

$$R = 16\pi G\, T_{00}$$

Global characteristics of ANO vortex:

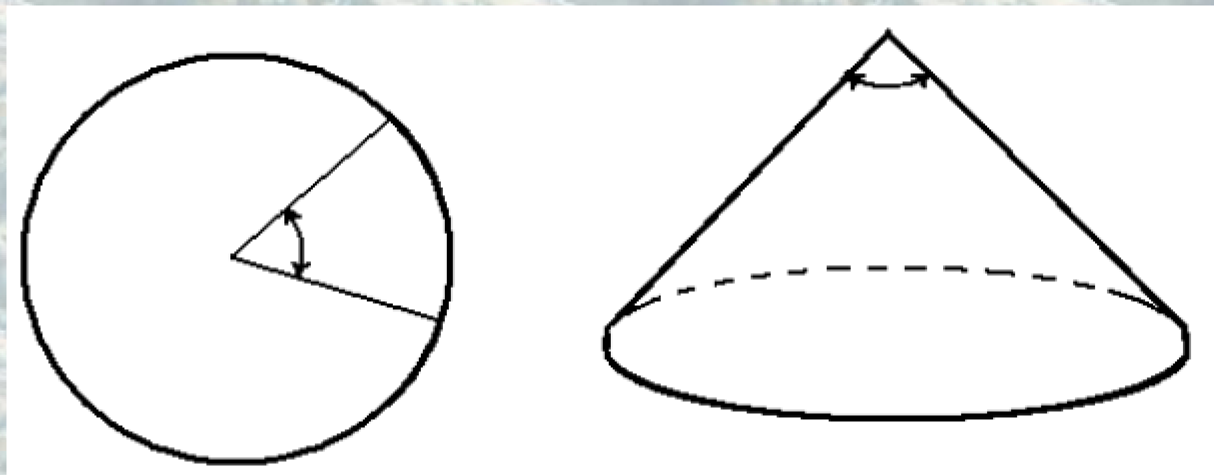
linear energy density: $$\mu = \frac{1}{16\pi G} \int_{core} d\sigma_{tr} R \sim m_H^2$$

flux of the gauge field: $$\Phi = \int_{core} d\sigma_{tr} B^3 = 2\pi n / g_H$$

The space-time metric outside the string core:

$$ds^2 = -dt^2 + dr^2 + (1-4G\mu)^2 r^2 d\varphi^2 + dz^2 = -dt^2 + dr^2 + r^2 d\tilde{\varphi}^2 + dz^2$$

where $0 \leq \varphi < 2\pi, \quad 0 \leq \tilde{\varphi} < 2\pi(1-4G\mu)$, a deficit angle is equal to $8\pi G\mu$



Cosmic strings were introduced in

T.W.B.Kibble, J. Phys. A 9, 1387 (1976); Phys. Rep. 67, 183 (1980)
A.Vilenkin, Phys. Rev. D 23, 852 (1981); D 24, 2082 (1981)

Earlier studies in

M.Fierz (unpublished)
J.Weber, J.A.Wheeler, Rev.Mod.Phys. 29, 509 (1957)
L.Marder, Proc. Roy. Soc. London A 252, 45 (1959)

A cosmic string resulting from a phase transition at the scale of the grand unification of all interactions is characterized by the values of the deficit angle

$$8\pi G\mu \sim (10^{-6} \div 10^{-5})$$

Generalization:

d+1-dimensional space-time with a codimension-2 brane

$$(r, \varphi, \overrightarrow{z}_{d-2})$$

ANO vortex

$$\phi = \frac{1}{\sqrt{2}}\sigma\tau(r)e^{in\varphi}$$

$$V_\varphi = \frac{n}{g_H}\tau^2(r)$$

$$B^{3..d} = \frac{2n}{g_H\, r}\tau\tau'$$

$$(V_r = \overrightarrow{V}_{d-2} = V_0 = 0)$$

stress-energy tensor

$$T_{00} = -T_{33} = \ldots = -T_{dd}$$

scalar curvature

$$R = 8\pi(d-1)G\,T_{00}$$

metric

$$ds^2 = -dt^2 + dr^2 + (1-4G\mu)^2 r^2 d\varphi^2 + d\vec{z}^{\,2}_{d-2}$$

tension

$$\mu = \frac{1}{8\pi(d-1)G}\int_{core} d\sigma_{tr} R$$

flux

$$\Phi = \int_{core} d\sigma_{tr} B^{3\ldots d}$$

Second quantized charged scalar field

$$\Psi(\mathbf{x},t) = \sum\!\!\!\!\!\!\int_\lambda \frac{1}{\sqrt{2E_\lambda}} \left[e^{-iE_\lambda t}\psi_\lambda(\mathbf{x})a_\lambda + e^{iE_\lambda t}\psi_{-\lambda}(\mathbf{x})b^\dagger_\lambda\right]$$

where $E_\lambda = E_{-\lambda} > 0$

$$[a_\lambda, a^\dagger_{\lambda'}]_- = [b_\lambda, b^\dagger_{\lambda'}]_- = \left\langle \lambda | \lambda' \right\rangle$$

$$a_\lambda|\text{vac}\rangle = b_\lambda|\text{vac}\rangle = 0$$

$$(-\boldsymbol{\nabla}^2 + m^2)\psi_\lambda(\mathbf{x}) = E^2_\lambda \psi_\lambda(\mathbf{x})$$

Vacuum current:

$$\mathbf{j}(\mathbf{x}) = \frac{1}{4i}\langle\text{vac}| \left\{\left[\Psi^\dagger(\mathbf{x},t),\ \boldsymbol{\nabla}\Psi(\mathbf{x},t)\right]_+ - \left[\boldsymbol{\nabla}\Psi^\dagger(\mathbf{x},t),\ \Psi(\mathbf{x},t)\right]_+\right\}|\text{vac}\rangle =$$

$$= \frac{1}{2i}\sum\!\!\!\!\!\!\int_\lambda E^{-1}_\lambda \left\{\psi^*_\lambda(\mathbf{x})[\boldsymbol{\nabla}_\lambda\psi_\lambda(\mathbf{x})] - [\boldsymbol{\nabla}\psi_\lambda(\mathbf{x})]^*\psi_\lambda(\mathbf{x})\right\}$$

Solution to the Klein–Gordon equation

$$\psi_{knp}(\mathbf{x}) = \frac{1}{2\pi\sqrt{1-\eta}} J_{\frac{|n-\Phi|}{1-\eta}}(kr) e^{in\varphi} e^{ipz}$$

where $0 < k < \infty$, $n \in \mathbb{Z}$, $-\infty < p < \infty$

$$V_r = 0, \qquad V_\varphi = \Phi$$

$$\int d^3x \sqrt{g}\, \psi^*_{knp}(\mathbf{x})\, \psi_{k'n'p'}(\mathbf{x}) = \frac{\delta(k-k')}{k} \delta_{nn'}\, \delta(p-p')$$

Vacuum current

$j_r = j_3 = 0$

$$j_\varphi(r) = \frac{1}{(2\pi)^2(1-\eta)} \int\limits_{-\infty}^{\infty} dp \int\limits_0^{\infty} dk\, k(p^2+k^2+m^2)^{-\frac{1}{2}} \sum_{n\in\mathbb{Z}} (n-\Phi) J^2_{\frac{|n-\Phi|}{1-\eta}}(kr)$$

periodic dependence on Φ (Bohm–Aharonov effect)

$$\Phi = [\![\Phi]\!] + F$$

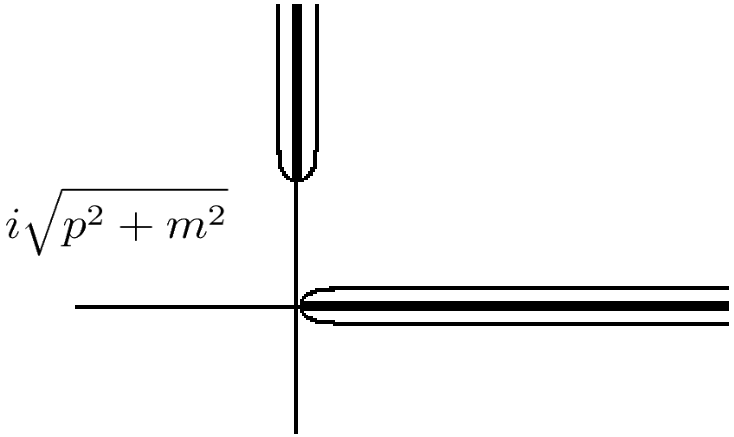
$[\![\Phi]\!]$ is the integer part of Φ

$$0 < F < 1$$

Calculation of integrals and a sum

1. $J_\rho^2(kr) = \frac{1}{i\pi}[I_\rho(-ikr)K_\rho(-ikr) - I_\rho(ikr)K_\rho(ikr)]$

$$\frac{2}{\pi}\int_{\sqrt{p^2+m^2}}^{\infty} d\kappa\,\kappa(\kappa^2-p^2-m^2)^{\frac{1}{2}}\sum_{n\in\mathbb{Z}}(n-\Phi)I_{\frac{|n-\Phi|}{1-\eta}}(\kappa r)K_{\frac{n-\Phi}{1-\eta}}(\kappa r)$$

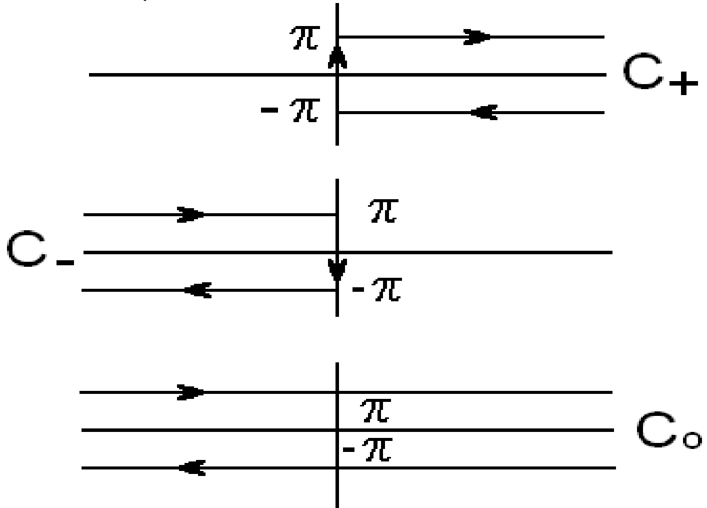


$$I_\rho(\kappa r)K_\rho(\kappa r) = \frac{1}{2}\int_0^\infty \frac{dy}{y}\exp\left(-\frac{\kappa^2 r^2}{2y} - y\right) I_\rho(y)$$

$$\sum_{l=1}^{\infty}(l-F)I_{\frac{l-F}{1-\eta}}(y) - \sum_{l=0}^{\infty}(l+F)I_{\frac{l+F}{1-\eta}}(y)$$

2. Schläfli contour representation

$$I_\rho(y) = \frac{1}{2\pi i}\int_{C_+} dz\, e^{y ch z - \rho z} = -\frac{1}{2\pi i}\int_{C_-} dz\, e^{y ch z + \rho z}$$



$$\sum_{l=1}^{\infty}(l-F)I_{a(l-F)}(y) - \sum_{l=0}^{\infty}(l+F)I_{a(l+F)}(y) =$$

$$= -\frac{y}{a}\frac{1}{4\pi i}\int_{C_0} dz\, e^{y ch z - a(F-\frac{1}{2})z}\frac{sh z}{sh(\frac{a}{2}z)}$$

$$a = \frac{1}{1-\eta}$$

Vacuum current induced by a codimension-2 brane

$$j_\varphi(r) = -\frac{32}{(4\pi)^{(d+3)/2}}\frac{m^{(d+1)/2}}{r^{(d-3)/2}}\int_1^\infty \mathrm{d}v\, v^{(1-d)/2} K_{(d+1)/2}(2mrv)\,\Lambda(v;\,F,\,\nu)$$

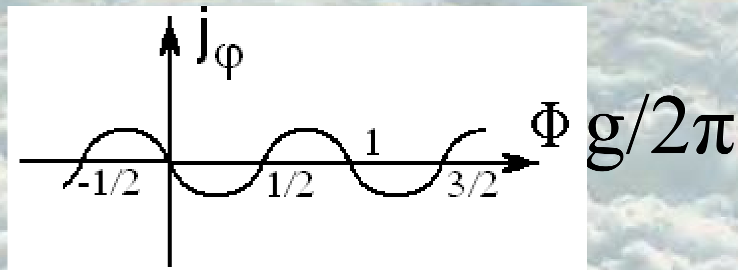
where

$$\Lambda(v;\,F,\,\nu) = \frac{\sin(F\nu\pi)\sinh[2(1-F)\nu\,\mathrm{arccosh}\,v] - \sin[(1-F)\nu\pi]\sinh(2F\nu\,\mathrm{arccosh}\,v)}{\cosh(2\nu\,\mathrm{arccosh}\,v) - \cos(\nu\pi)}$$

$$\nu = (1-4G\mu)^{-1}$$

$$F = \frac{g\Phi}{2\pi} - \left[\!\left[\frac{g\Phi}{2\pi}\right]\!\right]$$

$$F \to 1-F: \qquad j_\varphi \to -j_\varphi \qquad\qquad j_\varphi|_{F=\frac{1}{2}} = 0$$



Yu.A.S., N.D.Vlasii, Fortschr.Phys. 57, 705 (2009)

Induced vacuum magnetic field

Maxwell equation

$$\frac{1}{(d-2)!}\nabla_{i_2}\varepsilon^{i_1 i_2}{}_{i_3\ldots i_d}\, B_{(\mathrm{I})}^{i_3\ldots i_d} = e j^{i_1}$$

magnetic field strength

$$B_{(\mathrm{I})}^{3\ldots d}(r) = -\frac{16e\nu}{(4\pi)^{(d+3)/2}}\left(\frac{m}{r}\right)^{(d-1)/2}\int_1^{\infty}\mathrm{d}v\, v^{-(d+1)/2} K_{(d-1)/2}(2mrv)\,\Lambda(v;\, F,\, \nu)$$

and its flux

$$\Phi^{(\mathrm{I})} = \int_0^{2\pi}\mathrm{d}\varphi\int_0^{\infty}\mathrm{d}r\,\frac{r}{\nu}\, B_{(\mathrm{I})}^{3\ldots d}(r)$$

$d = 2:$ $$\Phi^{(\mathrm{I})} = \frac{e}{6m}\left(F - \frac{1}{2}\right) F(1-F)\nu^2$$

$d = 3:$ $$\Phi^{(\mathrm{I})} = \frac{e}{6\pi}\left(F - \frac{1}{2}\right) F(1-F)\nu^2(-\ln m r_0)$$

$d \geq 4:$ $$\Phi^{(\mathrm{I})} = -\frac{2e\,\Gamma\left(\frac{d-3}{2}\right)}{(4\pi)^{(d+1)/2}} r_0^{3-d} \int\limits_1^\infty \mathrm{d}v\, v^{-d} \Lambda(v;\, F,\, \nu)$$

$d = 3:$

Identifying r_0 with the transverse size of the string core, then $r_0 \sim m_H^{-1}$ and

$$\Phi^{(I)} = \frac{e}{6\pi}\left(F - \frac{1}{2}\right) \frac{F(1-F)}{(1-4G\mu)^2} \ln\frac{m_H}{m} \qquad m_H > m$$

Yu.A.S., N.D.Vlasii, Class. Quantum Grav. 26, 195009 (2009)

Energy-momentum tensor of charged massive scalar field

$$T^{\mu\nu}(x) = \frac{1}{2}\left[\nabla^{\mu}\Psi^{\dagger}(x), \nabla^{\nu}\Psi(x)\right]_{+} + \frac{1}{2}\left[\nabla^{\nu}\Psi^{\dagger}(x), \nabla^{\mu}\Psi(x)\right]_{+} +$$

$$+\frac{1}{4}g^{\mu\nu}\left[(\Box + m^{2} + \xi R)\Psi^{\dagger}(x), \Psi(x)\right]_{+} + \frac{1}{4}g^{\mu\nu}\left[\Psi^{\dagger}(x), (\Box + m^{2} + \xi R)\Psi(x)\right]_{+} +$$

$$+(\xi - \frac{1}{4})g^{\mu\nu}\Box\left[\Psi^{\dagger}(x), \Psi(x)\right]_{+} - \xi\left(\nabla^{\mu}\nabla^{\nu} + R^{\mu\nu}\right)\left[\Psi^{\dagger}(x), \Psi(x)\right]_{+},$$

point-splitting regularization

$$T^{\mu\nu}(x,x') = D^{\mu\nu}(x,x')\left[T\Psi(x)\Psi^{\dagger}(x') + T\Psi(x')\Psi^{\dagger}(x)\right],$$

where

$$D^{\mu\nu}(x,x') = \left(\frac{1}{2} - \xi\right)\left(\nabla_{x}^{\mu}\nabla_{x'}^{\nu} + \nabla_{x}^{\nu}\nabla_{x'}^{\mu}\right) + \xi g^{\mu\nu}(\Box_{x} + m^{2} + \xi R_{x}) + \xi g^{\mu\nu}(\Box_{x'} + m^{2} + \xi R_{x'}) +$$

$$+2\left(\xi - \frac{1}{4}\right)g^{\mu\nu}\left(g_{\rho\sigma}\nabla_{x}^{\rho}\nabla_{x'}^{\sigma} - m^{2} - \frac{1}{2}\xi R_{x} - \frac{1}{2}\xi R_{x'}\right) - \xi\left(\nabla_{x}^{\mu}\nabla_{x}^{\nu} + \nabla_{x'}^{\mu}\nabla_{x'}^{\nu} + \frac{1}{2}R_{x}^{\mu\nu} + \frac{1}{2}R_{x'}^{\mu\nu}\right).$$

Equation for Green's function

$$\left(\Box_x + m^2 + \xi R_x\right) i\langle 0 | T\Psi(x)\Psi^\dagger(x') | 0\rangle =$$

$$= \left(\Box_{x'} + m^2 + \xi R_{x'}\right) i\langle 0 | T\Psi(x)\Psi^\dagger(x') | 0\rangle = \frac{1}{\sqrt{g}}\delta(x - x')$$

decomposition of the VEV of the chronological operator product

$$\langle 0 | T\Psi(x)\Psi^\dagger(x') | 0\rangle = \langle 0 | T\Psi(x)\Psi^\dagger(x') | 0\rangle_S + \langle 0 | T\Psi(x)\Psi^\dagger(x') | 0\rangle_R .$$

Renormalized VEV of the energy-momentum tensor

$$t^{\mu\nu}(x) = \lim_{x' \to x} D^{\mu\nu}(x,x')\langle 0 | T\Psi(x)\Psi^\dagger(x') + T\Psi(x')\Psi^\dagger(x) | 0\rangle_R .$$

Induced vacuum energy-momentum tensor in the cosmic string background
Yu.A.S., Vlasii. Class. Quantum Grav. 29, 095002 (2012)

$$t_0^0(r) = t_3^3(r) = \frac{2\nu}{(2\pi)^3}\left(\frac{m}{r}\right)^2 \int_1^\infty \frac{d\,\mathrm{v}}{\sqrt{\mathrm{v}^2-1}} \left\{ \left[\mathrm{v}^{-2} + 2(1-4\xi) \right] K_2(2mr\,\mathrm{v}) - 2(1-4\xi) mr\,\mathrm{v}\, K_3(2mr\,\mathrm{v}) \right\} \Lambda(\mathrm{v};F,\nu),$$

$$t_1^1(r) = \frac{2\nu}{(2\pi)^3}\left(\frac{m}{r}\right)^2 \int_1^\infty \frac{d\,\mathrm{v}}{\sqrt{\mathrm{v}^2-1}} (\mathrm{v}^{-2} - 4\xi) K_2(2mr\,\mathrm{v}) \Lambda(\mathrm{v};F,\nu),$$

$$t_2^2(r) = \frac{2\nu}{(2\pi)^3}\left(\frac{m}{r}\right)^2 \int_1^\infty \frac{d\,\mathrm{v}}{\sqrt{\mathrm{v}^2-1}} (\mathrm{v}^{-2} - 4\xi) \left[K_2(2mr\,\mathrm{v}) - 2mr\,\mathrm{v}\, K_3(2mr\,\mathrm{v}) \right] \Lambda(\mathrm{v};F,\nu),$$

where

$$\Lambda(\mathrm{v};F,\nu) = \frac{\sin(F\nu\pi)\cosh[2(1-F)\nu \operatorname{arccosh} \mathrm{v}] + \sin[(1-F)\nu\pi]\cosh(2F\nu \operatorname{arccosh} \mathrm{v})}{\cosh(2\nu \operatorname{arccosh} \mathrm{v}) - \cos(\nu\pi)},$$

$$F = \frac{g\Phi}{2\pi} - \left[\!\left[\frac{g\Phi}{2\pi} \right]\!\right] \qquad \nu = (1-4G\mu)^{-1}$$

Conservation:

$$\nabla_{\mu} t^{\mu}_{\mu'} = 0$$

$$\partial_r t^1_1(r) + r^{-1}\left[t^1_1(r) - t^2_2(r)\right] = 0$$

Trace:

$$g_{\mu\mu'} t^{\mu\mu'} = \frac{\nu}{\pi^3}\frac{m}{r}\int_1^{\infty}\frac{d\,\mathrm{v}}{\mathrm{v}\sqrt{\mathrm{v}^2-1}}\left\{(1-6\xi)\,\mathrm{v}\,\frac{m}{r}\left[K_2(2mr\,\mathrm{v}) - mr\,\mathrm{v}\,K_3(2mr\,\mathrm{v})\right] - \frac{1}{2}m^2 K_1(2mr\,\mathrm{v})\right\}\Lambda(\mathrm{v};F,\nu)$$

conformal invariance is achieved in the massless limit at $\xi = \frac{1}{6}$:

$$\lim_{m\to 0} g_{\mu\mu'} t^{\mu\mu'}\Big|_{\xi=1/6} = 0.$$

For a global string

$$t_0^0(r)\,|_{F=0} = t_3^3(r)\,|_{F=0} = \frac{\nu \sin(\nu\pi)}{(2\pi)^3}\left(\frac{m}{r}\right)^2 \int_1^\infty \frac{d\,\mathrm{v}}{\sqrt{\mathrm{v}^2-1}}\,[\cosh^2(\nu\,\mathrm{arccosh}\,\mathrm{v}) - \cos^2(\nu\pi/2)]^{-1} \times$$

$$\times\left\{[\mathrm{v}^{-2} + 2(1-4\xi)]K_2(2mr\,\mathrm{v}) - 2(1-4\xi)mr\,\mathrm{v}\,K_3(2mr\,\mathrm{v})\right\},$$

$$t_1^1(r)\,|_{F=0} = \frac{\nu \sin(\nu\pi)}{(2\pi)^3}\left(\frac{m}{r}\right)^2 \int_1^\infty \frac{d\,\mathrm{v}}{\sqrt{\mathrm{v}^2-1}}\,\frac{\mathrm{v}^{-2} - 4\xi}{\cosh^2(\nu\,\mathrm{arccosh}\,\mathrm{v}) - \cos^2(\nu\pi/2)}\,K_2(2mr\,\mathrm{v}),$$

$$t_2^2(r)\,|_{F=0} = \frac{\nu \sin(\nu\pi)}{(2\pi)^3} \int_1^\infty \frac{d\,\mathrm{v}}{\sqrt{\mathrm{v}^2-1}}\,\frac{\mathrm{v}^{-2} - 4\xi}{\cosh^2(\nu\,\mathrm{arccosh}\,\mathrm{v}) - \cos^2(\nu\pi/2)}\left[K_2(2mr\,\mathrm{v}) - 2mr\,\mathrm{v}\,K_3(2mr\,\mathrm{v})\right]$$

For a vanishing string tension

$$t_0^0(r)\,|_{\nu=1} = t_3^3(r)\,|_{\nu=1} = \frac{2\sin(F\pi)}{(2\pi)^3}\left(\frac{m}{r}\right)^2 \int_1^\infty \frac{d\,\mathrm{v}}{\mathrm{v}\sqrt{\mathrm{v}^2-1}} \Big\{ \left[\mathrm{v}^{-2} + 2(1-4\xi)\right] K_2(2mr\,\mathrm{v}) -$$

$$-2(1-4\xi)mr\,\mathrm{v}\,K_3(2mr\,\mathrm{v})\Big\}\cosh[(2F-1)\,\mathrm{arccosh}\,\mathrm{v}],$$

$$t_1^1(r)\,|_{\nu=1} = \frac{2\sin(F\pi)}{(2\pi)^3}\left(\frac{m}{r}\right)^2 \int_1^\infty \frac{d\,\mathrm{v}}{\mathrm{v}\sqrt{\mathrm{v}^2-1}} (\mathrm{v}^{-2} - 4\xi) K_2(2mr\,\mathrm{v}) \cosh[(2F-1)\mathrm{arccosh}\,\mathrm{v}],$$

$$t_2^2(r)\,|_{\nu=1} = \frac{2\sin(F\pi)}{(2\pi)^3}\left(\frac{m}{r}\right)^2 \int_1^\infty \frac{d\,\mathrm{v}}{\mathrm{v}\sqrt{\mathrm{v}^2-1}} (\mathrm{v}^{-2} - 4\xi)\left[K_2(2mr\,\mathrm{v}) - 2mr\,\mathrm{v}\,K_3(2mr\,\mathrm{v})\right]\cosh[(2F-1)\mathrm{arccosh}\,\mathrm{v}].$$

Yu. A. Sitenko and V. M. Gorkavenko, Phys. Rev. D **67**, 085015 (2003).
Yu. A. Sitenko and A. Yu. Babansky, Mod. Phys. Lett. A **13**, 379 (1998).
Yu. A. Sitenko and A. Yu. Babansky, Phys. At. Nucl. **61**, 1594 (1998).

For a vanishing mass of scalar field:

$$\lim_{m\to 0} t_{\mu}^{\mu'}(r) = \frac{1}{12\pi^2 r^4}\left(\frac{1}{60}\left\{1-\nu^4\left[1-30F^2(1-F)^2\right]\right\}\mathrm{diag}(1,1,-3,1)+\right.$$

$$\left.+\left(\xi-\frac{1}{6}\right)\left\{1-\nu^2[1-6F(1-F)]\right\}\mathrm{diag}(2,-1,3,2)\right)$$

Trace:

$$\lim_{m\to 0} g_{\mu\mu'}t^{\mu\mu'} = \frac{1}{2\pi^2 r^4}\left(\xi-\frac{1}{6}\right)\left\{1-\nu^2[1-6F(1-F)]\right\}$$

V. P. Frolov and E. M. Serebriany, Phys. Rev. D **35**, 3779 (1987).
J. S. Dowker, Phys. Rev. D **36**, 3742 (1987).

$d+1$ -dimensional space-time

$$t_0^0(r) = t_j^j(r) = \frac{16\nu}{(4\pi)^{(d+3)/2}}\left(\frac{m}{r}\right)^{(d+1)/2}\int_1^\infty d\,\mathrm{v}\,\frac{\mathrm{v}^{(3-d)/2}}{\sqrt{\mathrm{v}^2-1}}\times$$

$$\times\left\{[\mathrm{v}^{-2}+2(1-4\xi)]K_{(d+1)/2}(2mr\,\mathrm{v})-2(1-4\xi)mr\,\mathrm{v}\,K_{(d+3)/2}(2mr\,\mathrm{v})\right\}\Lambda(\mathrm{v};F,\nu),$$

$$t_1^1(r) = \frac{16\nu}{(4\pi)^{(d+3)/2}}\left(\frac{m}{r}\right)^{(d+1)/2}\int_1^\infty d\,\mathrm{v}\,\frac{\mathrm{v}^{(3-d)/2}}{\sqrt{\mathrm{v}^2-1}}(\mathrm{v}^{-2}-4\xi)K_{(d+1)/2}(2mr\,\mathrm{v})\Lambda(\mathrm{v};F,\nu),$$

$$t_2^2(r) = \frac{16\nu}{(4\pi)^{(d+3)/2}}\left(\frac{m}{r}\right)^{(d+1)/2}\int_1^\infty d\,\mathrm{v}\,\frac{\mathrm{v}^{(3-d)/2}}{\sqrt{\mathrm{v}^2+1}}(\mathrm{v}^{-2}-4\xi)\times$$

$$\times[K_{(d+1)/2}(2mr\,\mathrm{v})-2mr\,\mathrm{v}\,K_{(d+3)/2}(2mr\,\mathrm{v})]\Lambda(\mathrm{v};F,\nu),$$

where

$$\Lambda(\mathrm{v};F,\nu) = \frac{\sin(F\nu\pi)\cosh[2(1-F)\nu\,\mathrm{arccosh}\,\mathrm{v}]+\sin[(1-F)\nu\pi]\cosh(2F\nu\,\mathrm{arccosh}\,\mathrm{v})}{\cosh(2\nu\,\mathrm{arccosh}\,\mathrm{v})-\cos(\nu\pi)}$$

Transverse size of a codimension-2 brane

temporal component of the vacuum energy-momentum tensor
(vacuum energy density)

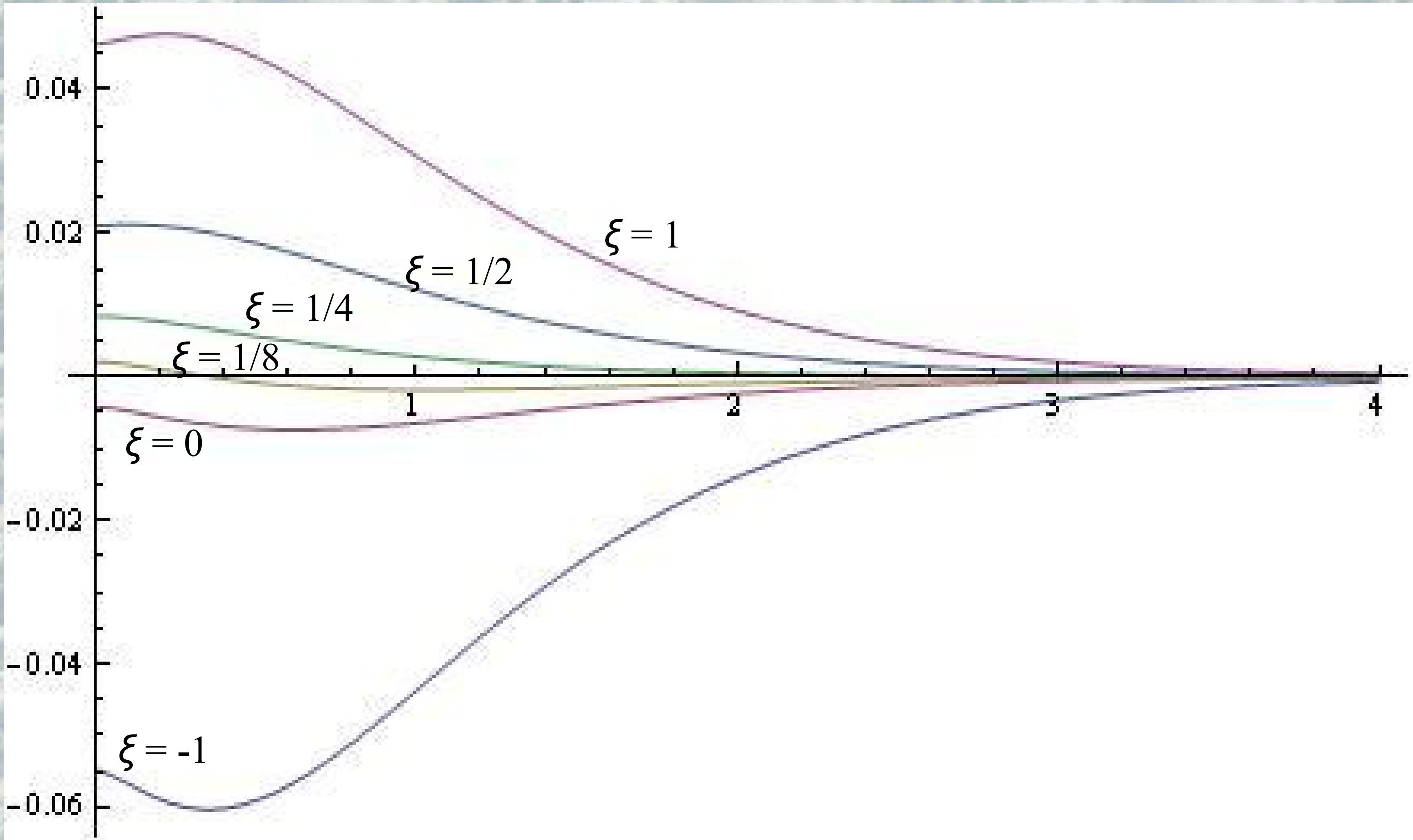
V.M.Gorkavenko,Yu.A.S., O.B.Stepanov. Intern.J.Mod.Phys.A 26, 3889 (2011)

$$t^{00} = \left\langle \text{vac} \,|\left[\partial_0 \Psi^{+} \partial_0 \Psi + \partial_0 \Psi \partial_0 \Psi^{+} - (\xi - 1/4) \Delta (\Psi^{+} \Psi + \Psi \Psi^{+}) \right]|\, \text{vac} \right\rangle$$

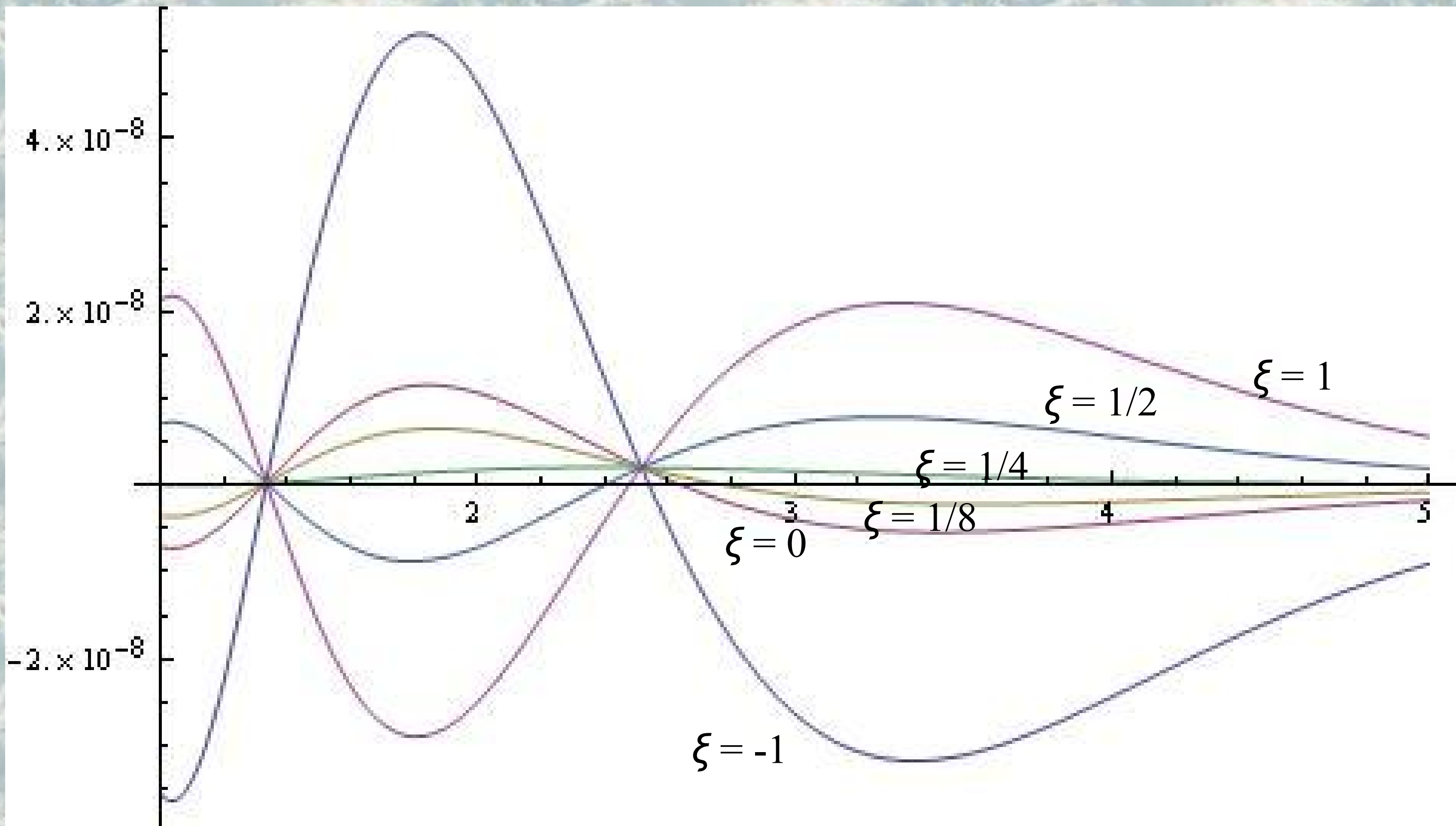
$$d = 2 \qquad \nu = 1$$

$$\frac{e\Phi}{2\pi} = n + \frac{1}{2}$$

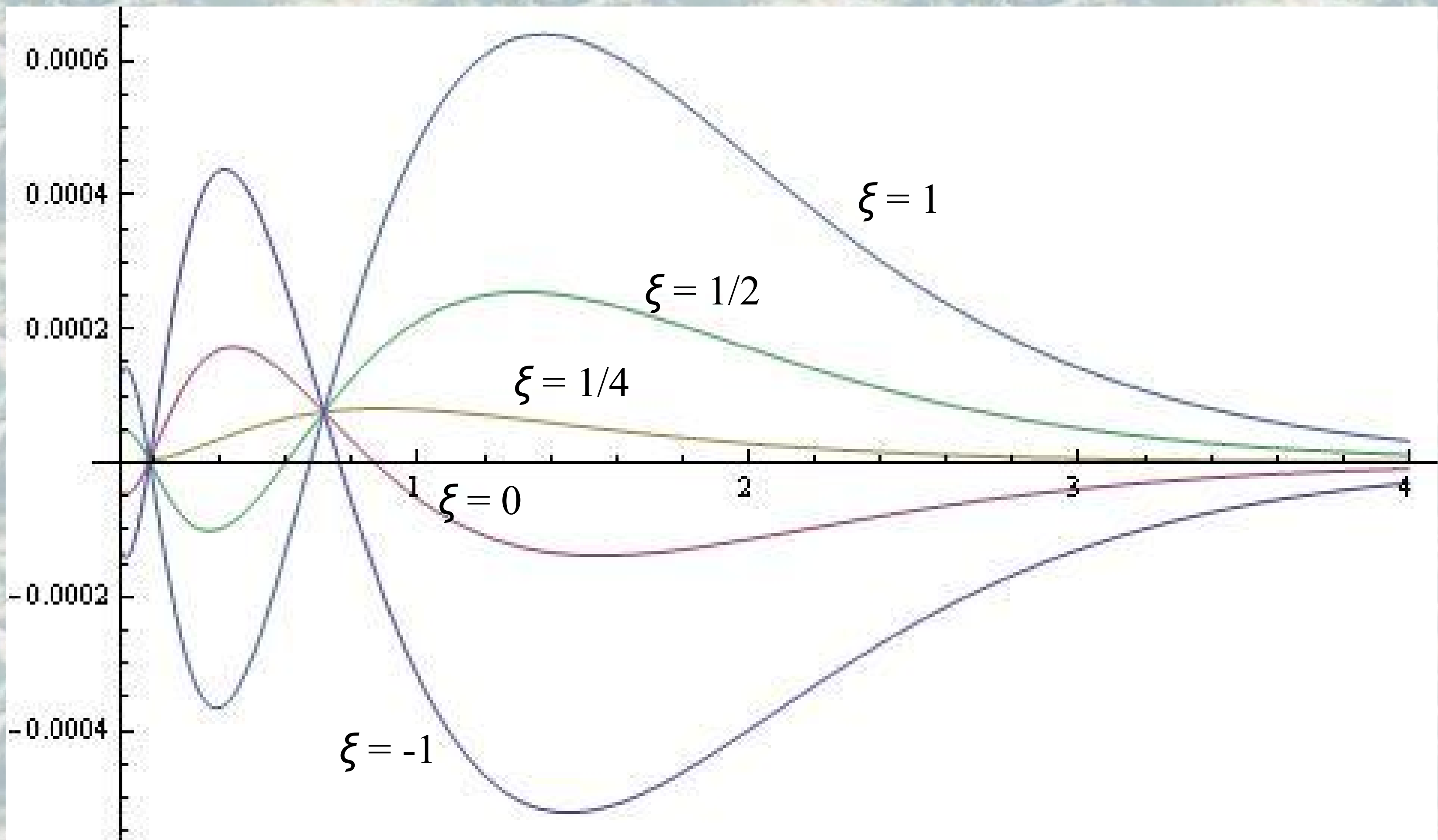
Dimensionless energy density $r^3 t^{00}$ as a function of mr (singular brane)



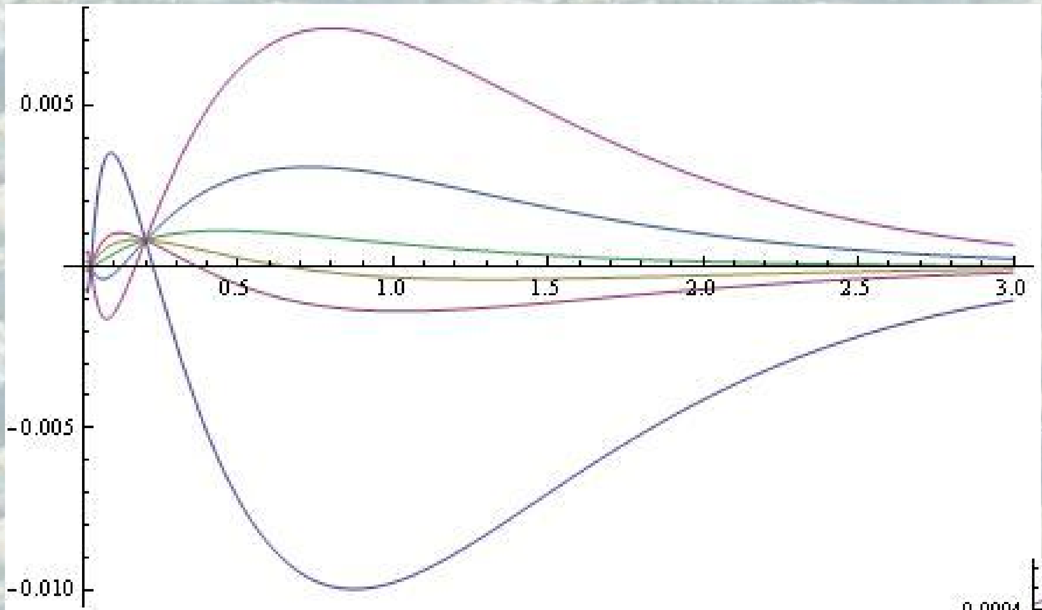
Dimensionless energy density $r^3 t^{00}$ as a function of mr (the case of $mr_0=1$)



Dimensionless energy density $r^3 t^{00}$ as a function of mr (the case of $mr_0 = 10^{-1}$)

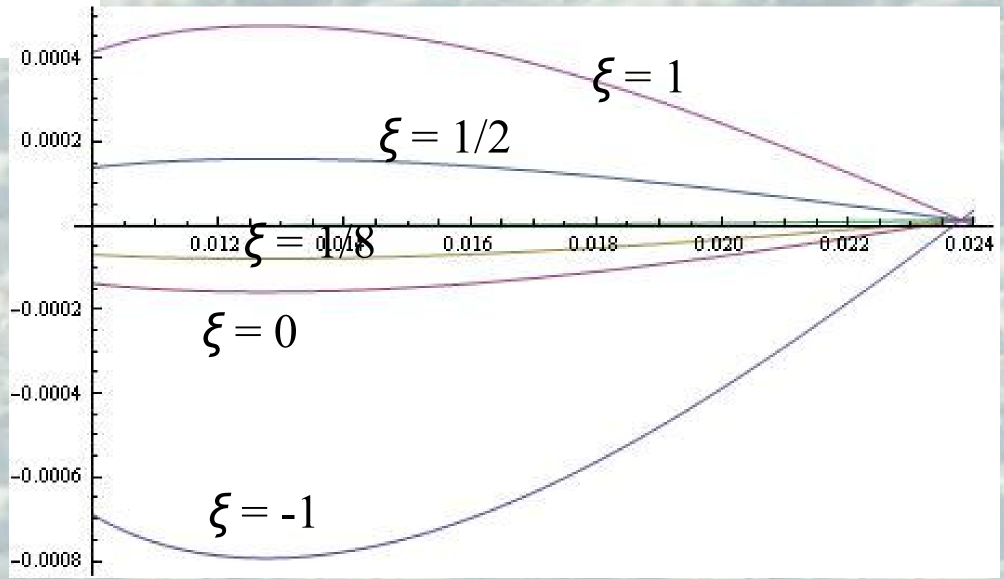


Dimensionless energy density $r^3 t^{00}$ as a function of mr (the case of $mr_0=10^{-2}$)



original

zoomed



Total energy

V.M.Gorkavenko,Yu.A.S., O.B.Stepanov. Intern.J.Mod.Phys.A 26, 3889 (2011)

$$E = 2\pi \int_0^\infty t^{00} r dr$$

$$E_{mr_0=1} = 6.944 \times 10^{-10}$$

$$E_{mr_0=1/10} = 0.000165$$

$$E_{mr_0=1/100} = 0.0106$$

Conclusions

- Cosmic string induces current and magnetic field in the vacuum
- Cosmic string induces energy-momentum tensor in the vacuum
- Generalization to arbitrary dimensions of space-time
- Finite thickness effects:
  - $\xi < 1/4$ is preferable
  - Casimir force acts to widen the string
  - vacuum energy is positive and independent of ξ

Thank you