Multiple ΛCDM cosmology with string landscape features and future singularities

Makarenko Andrey
Russia, Tomsk

Belgrade, 2012

The Dark Energy

Astronomical observations indicate that our Universe is currently in an accelerated phase.

This acceleration in the expansion rate of the observable cosmos is usually explained by introducing the so-called dark energy.
In the most common models considered in the literature, the dark energy comes from an ideal fluid with a specific equation of state (EoS) sometimes exhibiting rather strange properties, as a negative pressure and/or a negative entropy, also the fact that its action was invisible in the early universe while it is dominant in our epoch, etc.

According to the latest observational data, dark energy currently accounts for some 73% of the total mass-energy of the universe.

Models of Dark Energy

f(R)-gravity

$$S = \int d^4x \sqrt{-g}\,(f(R) + L_m)$$

Gauss-Bonnet gravity

$$S = \int d^4x \sqrt{-g}\left(\frac{1}{2\kappa^2}R + f(G) + L_m\right)$$

Theory with additional fields

$$S = \int d^4x \sqrt{-g}\left\{\frac{1}{2\kappa^2}R - \frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi) + L_{\mathrm{matter}}\right\}$$

Multidimensional theory

$$S = \int d^6x \sqrt{-g}(R + \epsilon L_{GB})$$

Non local theory

$$S = \int d^4x \sqrt{-g}\left\{\frac{1}{2\kappa^2}\left\{R\left(1 + f(\Box^{-1}R)\right) - 2\Lambda\right\} + \mathcal{L}_{\mathrm{matter}}(Q; g)\right\}$$

General Relativity with an ideal fluid can be rewritten, in an equivalent way, as some modified gravity. Also, the introduction of a fluid is to be seen as a phenomenological approach, since no explanation for the origin of such dark fluid is usually available.

The interesting possibility to explain the dark fluid origin may be related with string theory. The following sequence may be proposed: string/M-theory is approximated by modified (super)gravity which is observed finally as General Relativity with exotic dark fluid. If such conjecture is (even partially) true, it is expected that some string-related phenomena may be typical in dark energy universe. One celebrated stringy effect possibly related with early universe is string landscape which may lead to some observational consequences, since it could be responsible for the actual discrete mass spectrum of scalar and spinorial equations.

Ideal fluid

The equation of state (EoS) parameter w_{D} for dark energy is negative:

$$w_{\mathrm{D}} = p_{\mathrm{D}}/\rho_{\mathrm{D}} < 0$$

$$w = -1.04^{+0.09}_{-0.10}$$

$w > -1$ quintessence type

$w = -1$ deSitter type

$w < -1$ phantom type

The equations

For the spatially-flat FRW universe with metric

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} (dx^i)^2$$

the cosmological equations, that is, the FRW equations are given by

$$\frac{3}{\kappa^2} H^2 = \rho \,, \quad -\frac{2}{\kappa^2} \dot{H} = p + \rho \,.$$

We choose the pressure and energy density in the form

$$\rho = \frac{3}{\kappa^2} f(q)^2 \,, \quad p = -\frac{3}{\kappa^2} f(q)^2 - \frac{2}{\kappa^2} f'(q)$$

In case that $f'(q) = 0$ has a solution $q = q_0$, there is a solution where H is a constant

$$H = H_0 \equiv f(q_0) \ ,$$

where $\rho = -p$, what corresponds to an effective cosmological constant.

Then, if there is more than one solution satisfying

$$f'(q) = 0, \text{ as } q = q_n, \; n = 0, 1, 2, \cdots,$$

the theory could effectively admit several different cosmological constants, namely

$$H = f(q_n) , \quad \Lambda_n = 3 f(q_n)^2 .$$

Easy to get a solution of FRW equation

$$H = f(t)$$

Note that the origin of the time can be chosen arbitrary. t = q but one may choose t = q + t_0 with an arbitrary constant t_0. This shows that besides the solution H=f(t), H = f(t − t_0) can be a solution.

NON-PERIODIC BEHAVIOR OF THE DARK FLUID

Consider the simplest case with two values of the cosmological constant

$$\dot{H} = q(\Lambda_1 - t)(\Lambda_2 - t)(1+\beta t)^\gamma$$

In this case the Hubble parameter takes the form

$$H = q\frac{(1+\beta t)^{1+\gamma}}{\beta^3(1+\gamma)(2+\gamma)(3+\gamma)} \times \left(2+\beta\left((3+\gamma)(\Lambda_2+\Lambda_1(1+\beta\Lambda_2(2+\gamma))\right) - (1+\gamma)(2+\beta(\Lambda_1+\Lambda_2)(3+\gamma))t + \beta(1+\gamma)(2+\gamma)t^2\right)\right)$$

It is easy to find the form of the scale factor

$$a(t) = a_0 e^{q\frac{(1+\beta t)^{2+\gamma}\left(6+\beta\left((4+\gamma)(2\Lambda_2+\Lambda_1(2+\beta\Lambda_2(3+\gamma))\right)-(1+\gamma)(4+\beta(\Lambda_1+\Lambda_2)(4+\gamma))t+\beta(1+\gamma)(2+\gamma)t^2\right)\right)}{\beta^4(1+\gamma)(2+\gamma)(3+\gamma)(4+\gamma)}}$$

There are several different estimates of the cosmological parameters for current universe

H=13.6 Gyr

$$q_0 = -\left.\frac{1}{aH^2}\frac{d^2a}{dt^2}\right|_{t=t_0} = -\frac{1}{H^2}\left.\left\{\frac{1}{2}\frac{d\left(H^2\right)}{dN} + H^2\right\}\right|_{N=0} = -0.81 \pm 0.14$$

$$j_0 = \left\{\left.\frac{1}{aH^3}\frac{d^3a}{dt^3}\right|_{t=0} = \frac{1}{2H^2}\frac{d^2\left(H^2\right)}{dN^2} + \frac{3}{2H^2}\frac{d\left(H^2\right)}{dN} + 1\right\}\Bigg|_{N=0} = 2.16^{+0.81}_{-0.76}$$

NON-PERIODIC BEHAVIOR OF THE DARK FLUID

Suppose now that $\Lambda_1 = 0.1$ and $\Lambda_2 = 13.6$, at these points where we have an effective cosmological constant.

We choose, as an example of two parameter values for gamma: $\gamma = 12$ and $\gamma = -5$. In the first case, in order for the jerk parameter to be in the permissible region, it is necessary that the parameter β be in the range $0.00433706 < \beta < 0.00660997$. In the second case, we have that $-0.0228368 < \beta < -0.0164559$. Thus, for this choice of constants, we have the following values of the cosmological parameters:

$$j_0 = 2.16^{+0.81}_{-0.76}\,, \quad q_0 = -1\,, \quad H_0^{-1} = 13.6\,\mathrm{Gyr}\,, \quad w = -1\,.$$

Assume that $\Lambda_2 = 14$ ($t_0 = 13.6$). Then, the parameter β has to take values in the range: $0.00637252351 < \beta < 0.006847247$. Thus, for this choice of constant, we have the following values for the cosmological parameters:

$$2.452 < j_0 < 2.97\,, \quad -0.95 > q_0 > -0.932\,, \quad H_0^{-1} = 13.6\,\mathrm{Gyr}\,, \quad -0.967 < w < -0.955\,.$$

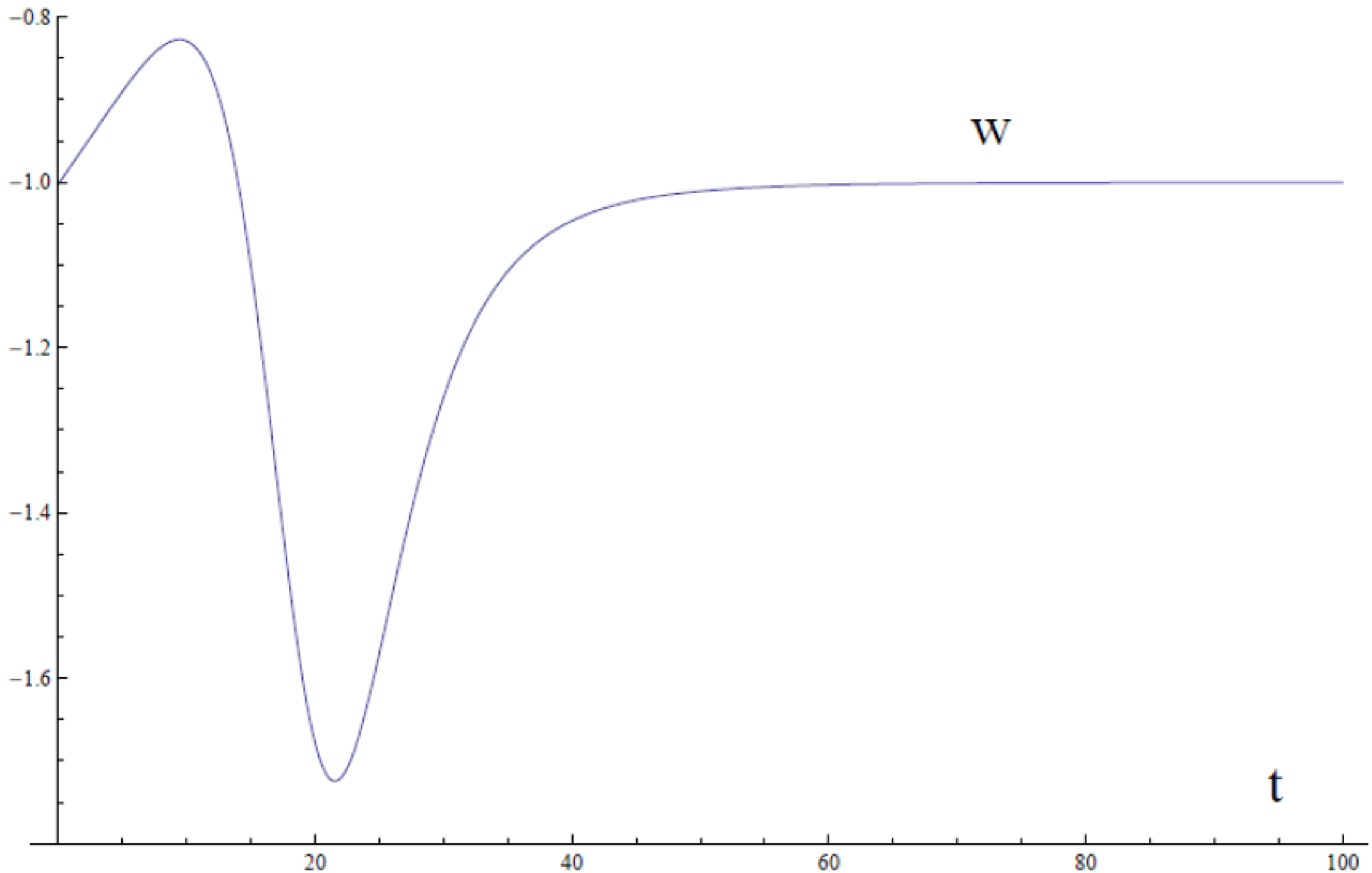
As we see, in this case $w > -1$, for $t \to \infty$ we have $w \to -1$.

In this case, for $t \to \infty$ we obtain that $H \to +\infty$ and we have a Little Rip singularity.

As is known, bound objects in such universe disintegrate. One can estimate the time required for the solar system disintegration, the dimensionless internal force being

$$F_{\mathrm{iner}} = \frac{\ddot{a}}{aH_0^2}$$

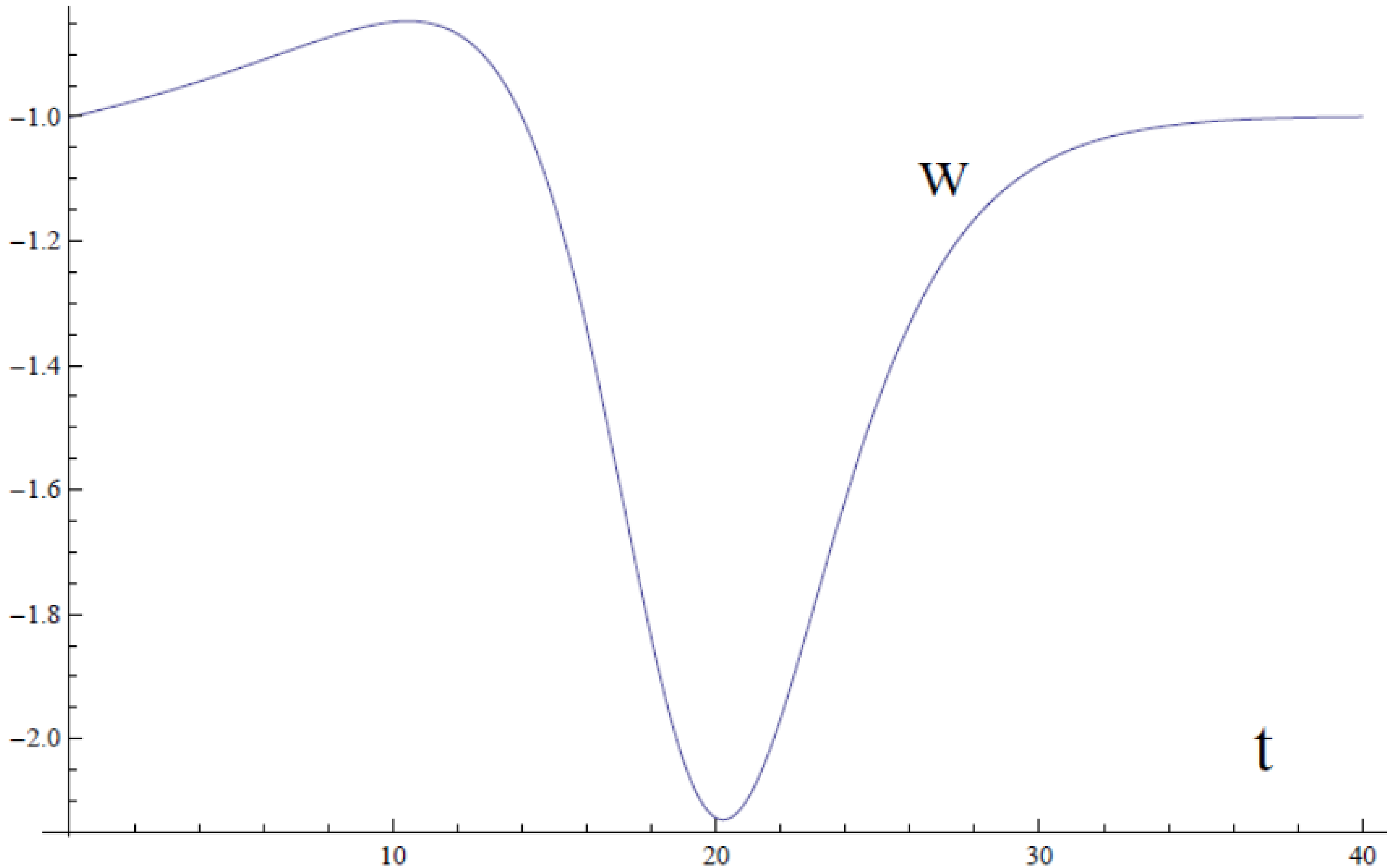
The Sun-Earth system disintegrates when $F_{iner} \sim 10^{23}$ and we find this time to be 563.58 Gyr (here Λ_1 = 0.1, Λ_2 = 14, β = 0.00637252351, γ = 12, and q = 0.0000184648)



Suppose now that $\gamma = -5$ and $\Lambda_2 = 14$ ($t_0 = 13.6$), then $-0.0222 > \beta > -0.02364$, and we have the following cosmological parameters:

$$2.3915 < j_0 < 2.45178\,, \quad -0.95 > q_0 > -0.92868\,, \quad H_0^{-1} = 13.6\,\mathrm{Gyr}\,, \quad -1.06942 < w < -1.0492\,.$$

If $\gamma < -4$ then we see that there is a singularity in the future at finite time (a Big Rip singularity), and $w \to -1$.
The lifetime of the universe that we find for the following values of the constants: $\Lambda_1 = 0.1$, $\Lambda_2 = 14$, $\beta = -0.023$, and $\gamma = -5$, $q = 9.329681063413538 \times 10^{-6}$, is $42.28 < t < 45.04$. In the same way one construct other examples of future evolution with Type II or Type III future singularity.



−1.0
−1.2
−1.4
−1.6
−1.8
−2.0
10
20
30
40
w
t

PERIODIC BEHAVIOR OF DARK FLUID

As a second example, slightly different from the one above, consider the ideal fluid:

$$f(t) = H = H_0 \mathrm{e}^{-g\left(t - \frac{1}{\omega}\sin\omega t\right)}$$

which yields

$$f'(t) = -H_0 g\,(1 - \cos\omega t)\,\mathrm{e}^{-g\left(1 - \frac{1}{\omega}\sin\omega t\right)}$$

Therefore, $f'(t) = 0$ when $t = 2\pi n/\omega$ for integer n. An effective multiple cosmological constant appears as

$$\Lambda_n = 3H_0^2 \mathrm{e}^{-\frac{4\pi n g}{\omega}}$$

Again, $t = 2\pi n/\omega$ corresponds to the cosmological constants and, therefore, the time-dependent solution could describe the transition between the cosmological constants, from the larger to the smaller one. In the limit of $t \to +\infty$ or $n \to +\infty$, the effective cosmological constant vanishes: $\lim_{n\to+\infty} \Lambda_n = 0$.

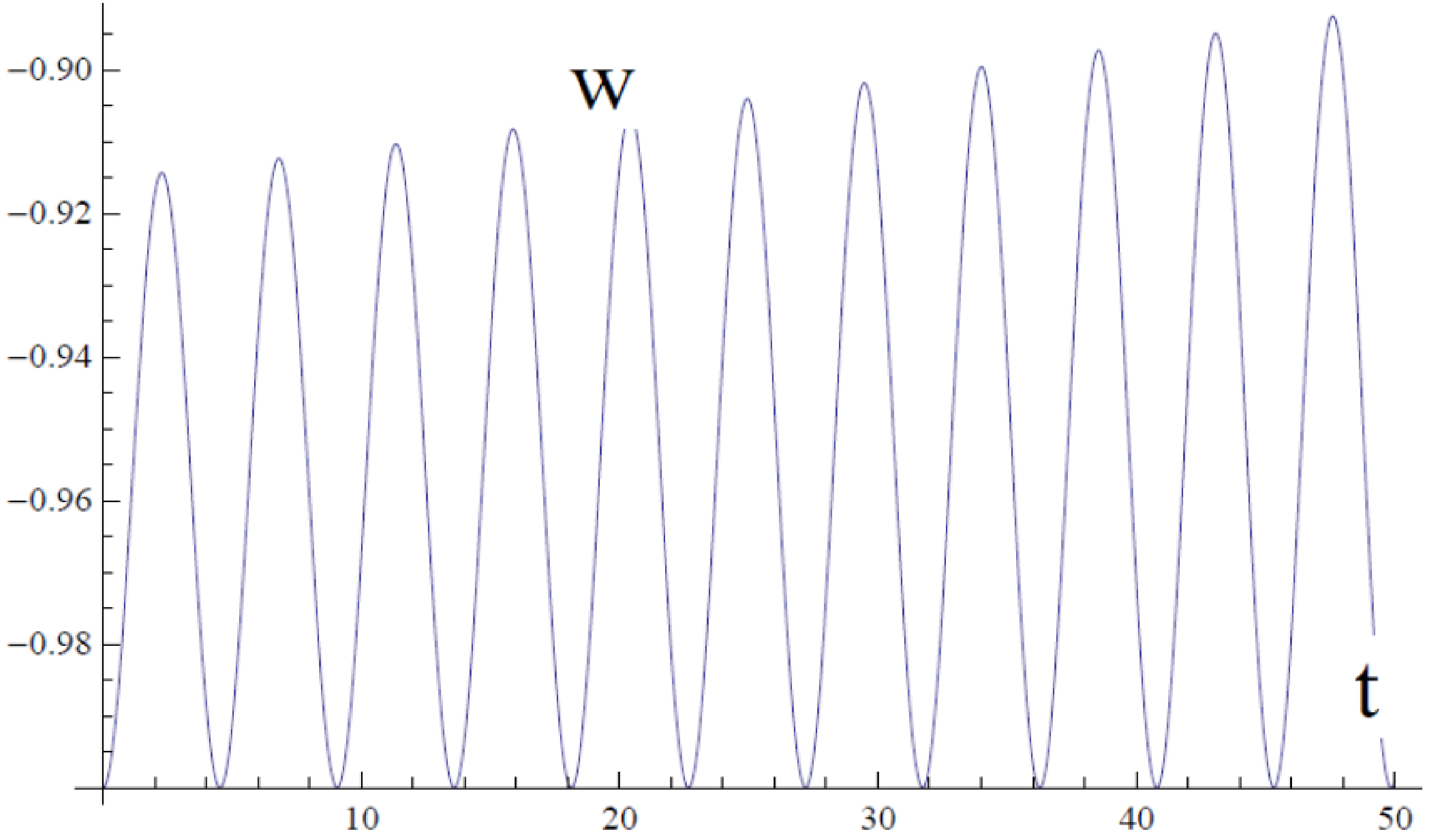
Now assume that, for $t = 13.6$ Gyr, the Hubble constant is 13.6^{-1} Gyr^{-1}. We choose the parameters:

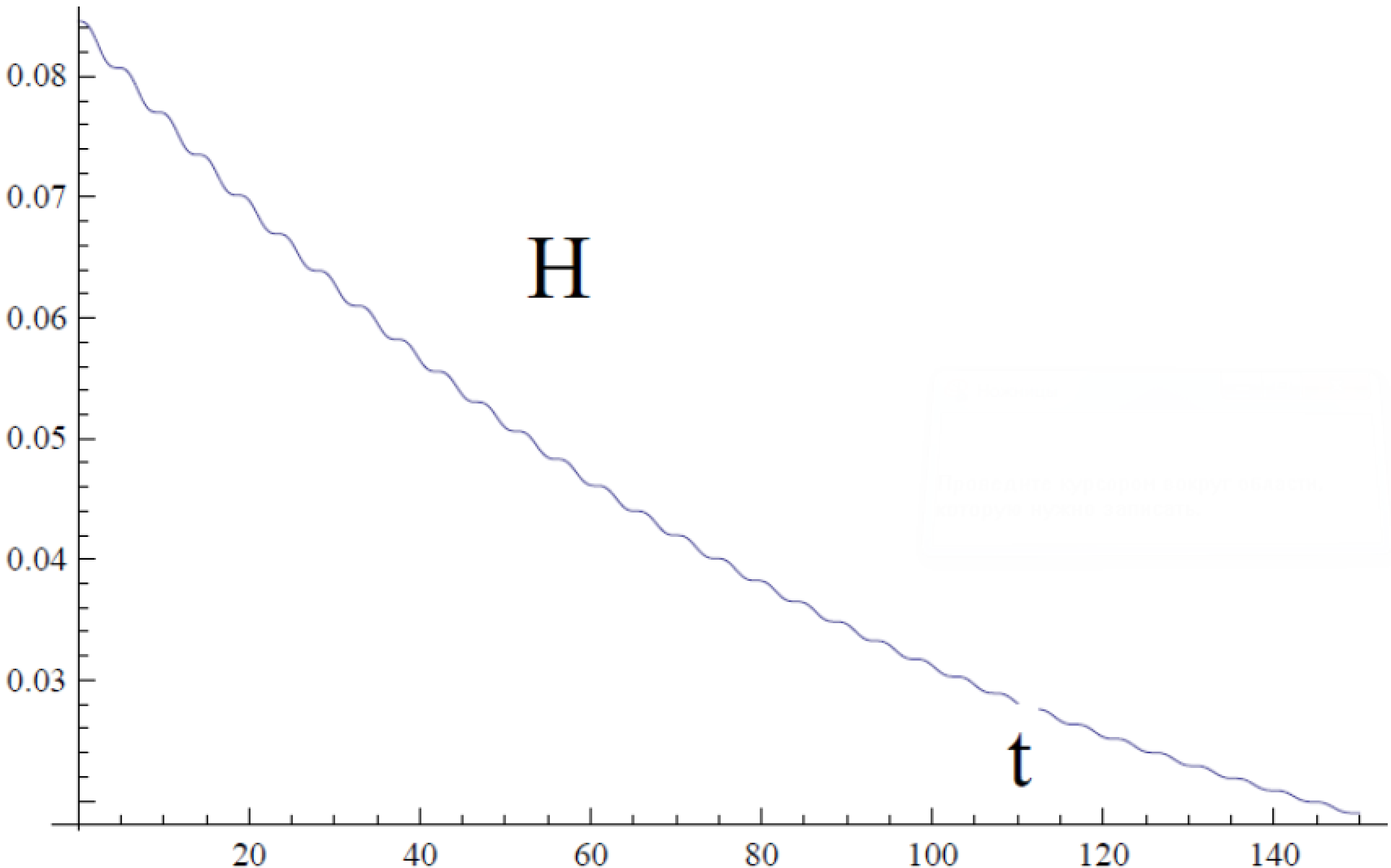
$$\omega = \frac{3\pi}{7}, \quad g = 0.01, \quad H_0 = 0.084579$$

for which we find the following values for the cosmological parameters:

$$j_0 = 2.21941\,, \quad q_0 = 0.980753\,, \quad H^{-1} = 13.6\,\mathrm{Gyr}\,, \quad w = -0.987168$$

And the universe is asymptotically de Sitter one.




H
t
0.08
0.07
0.06
0.05
0.04
0.03
20
40
60
80
100
120
140

We now consider the following choice for $f(q)$,

$$f(t) = H = \frac{q}{(1 + c_1\,(t - b\sin(ct)))^g}$$

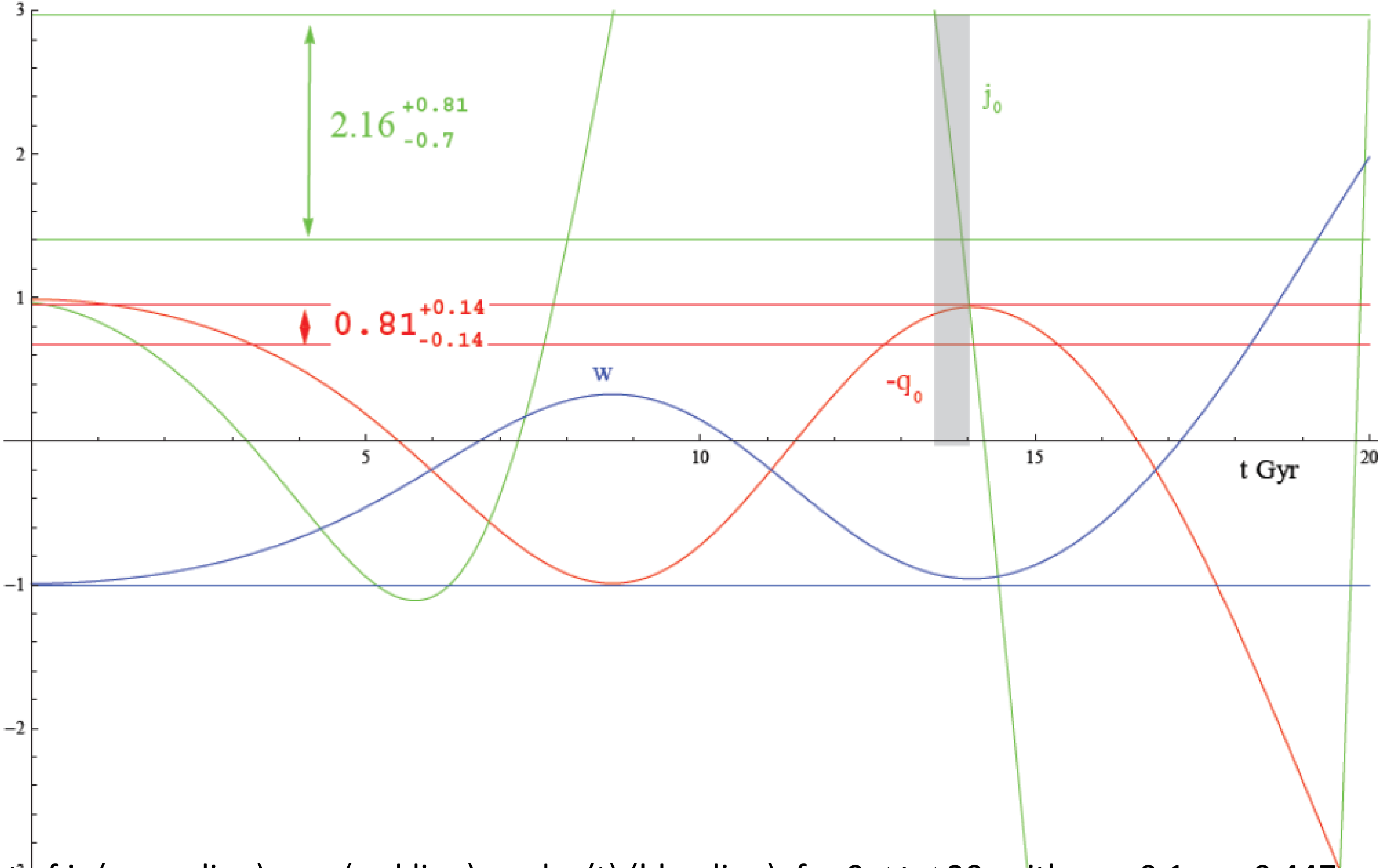
where c, c_1, q, b, and g are constants. Then,

$$\dot{H} = c_1 g q(-1 + bc\cos(ct))\,(1 + c_1 t - b\,c_1\sin(ct))^{-1-g}$$

and it is easy to see that the time derivative of the Hubble constant will vanish periodically ($b\,c\cos(ct) = 1$). We thus obtain a model with an effective cosmological constant $p = -\rho$. For the model, we have

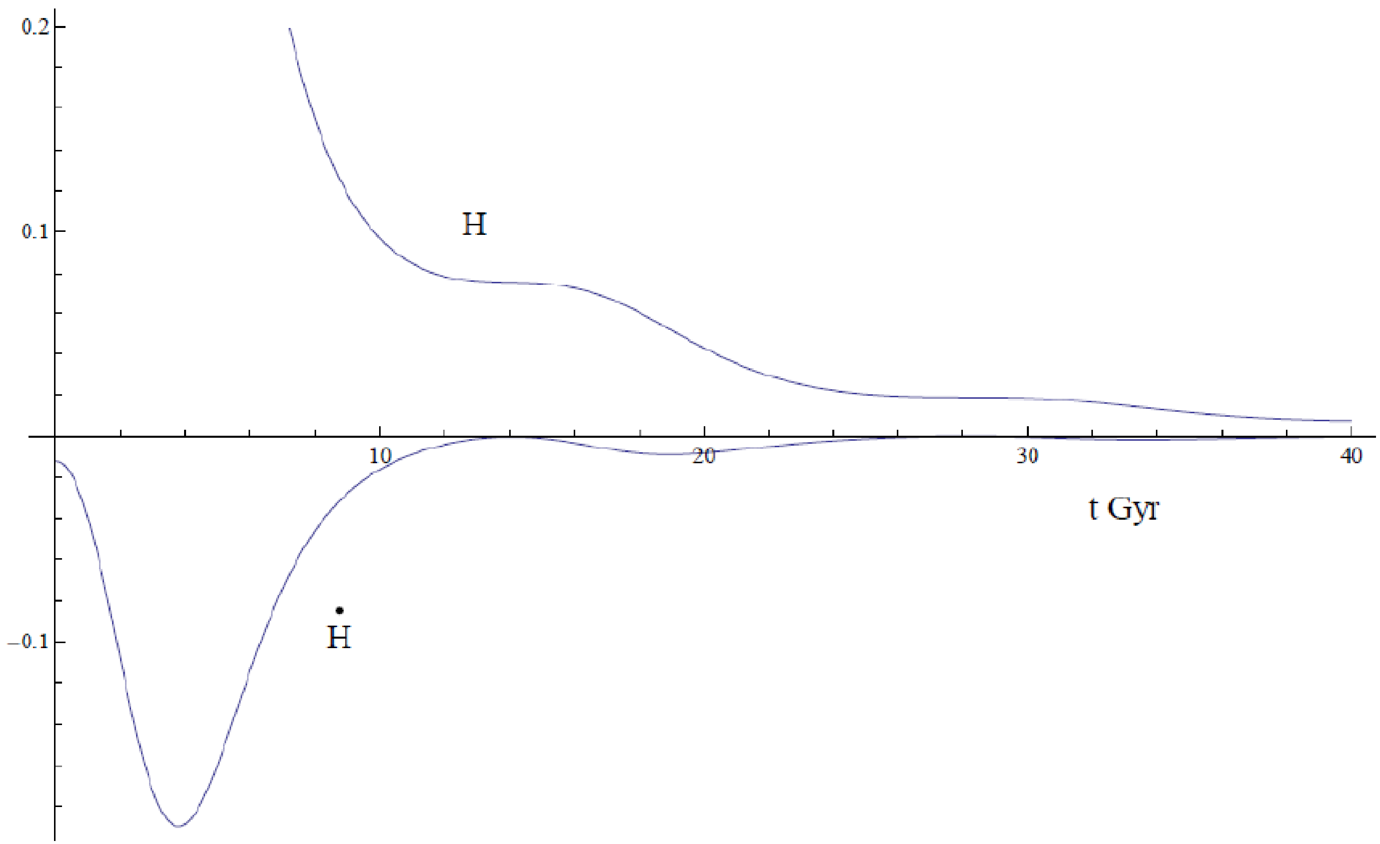
$$w_{\text{eff}} = -1 - \frac{2c_1 g\,(-1 + b\,c\,\cos(ct))\,(1 + c_1\,t - b\,c_1\sin(ct))^{-1+g}}{3q}$$

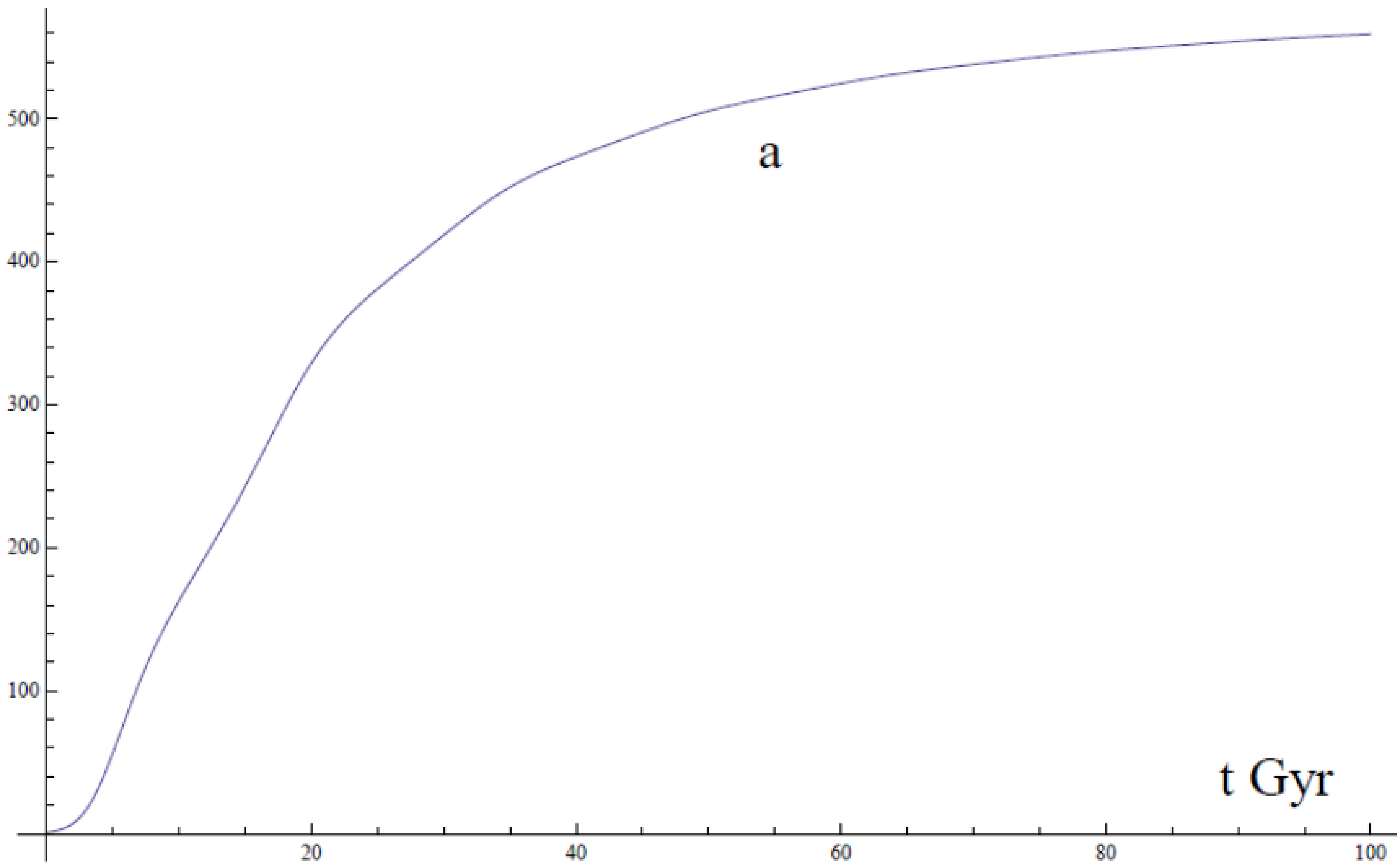
Note that there is a large arbitrariness in the choice of the constants, since one can choose them so that the parameters strictly match their current values, and one can provide the required stages of the universe evolution: Accelerating primordial universe ($-1/3 < w < -1$), deceleration of the universe ($-1/3 < w < 1/3$), and after that, when $w < -1/3$, the universe turns into an acceleration phase again. That is, a transition occurs from the accelerating to the decelerating phase, and back.



Plot of j_0 (green line), $-q_0$ (red line), and $w(t)$ (blue line), for $0 \le t \le 20$, with $c_1 = 0.1$, $c = 0.447$, $b = 2.15$, $q = 1.0445$, $g = 3$. The lines of constant time determine the range of possible values, for the current time, of these quantities. The highlighted time interval corresponds to the values of q_0 and j_0 at present.

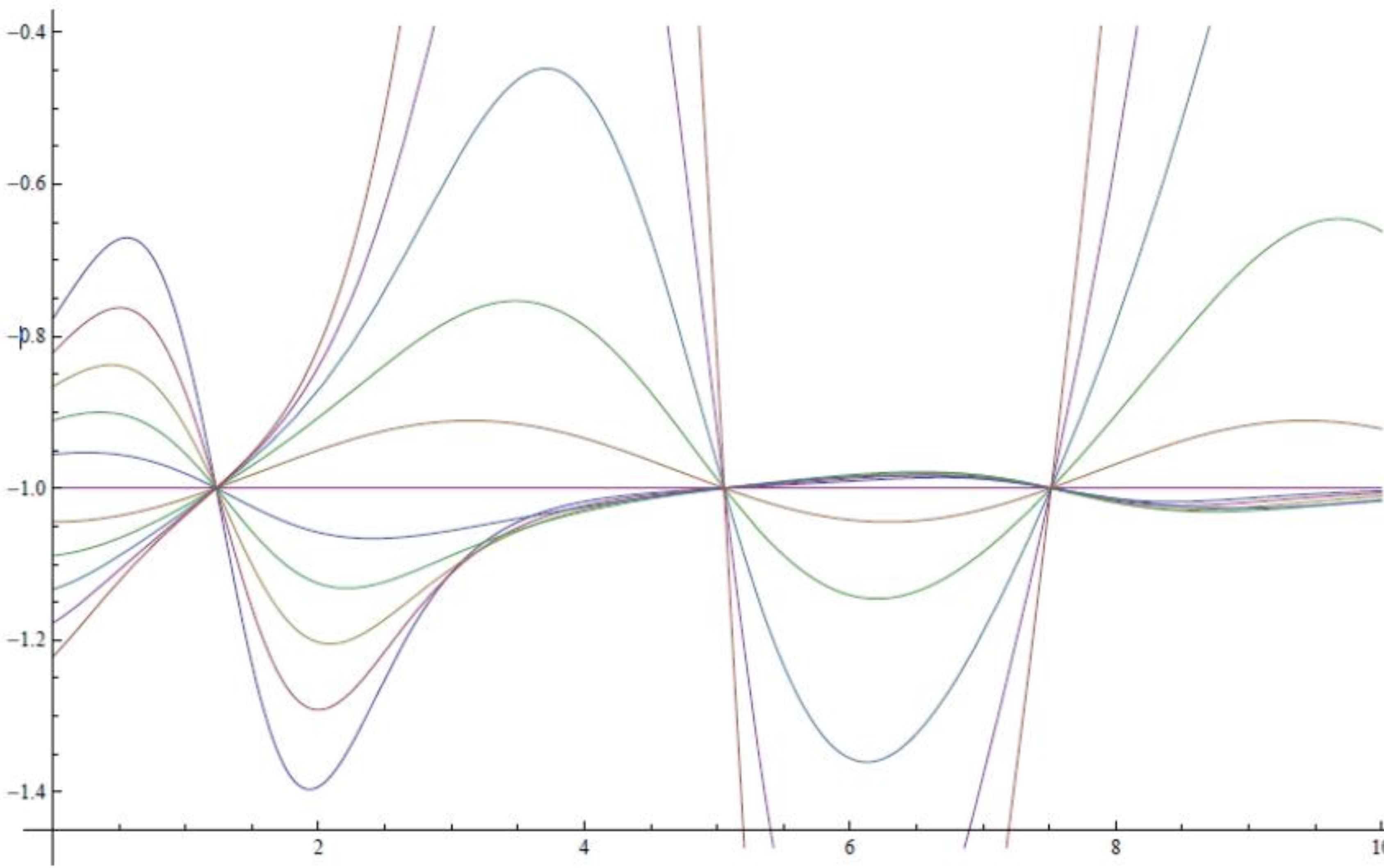
For $c_1 = 0.1$, $c = 0.447$, $b = 2.15$, $q = 1.0445$, $g = 3$, and $t = 13.6$ Gyr we find the following values of the cosmological parameters: $q_0 = -0.902$, $j_0 = 2.639$, $w = -0.935$, and $H_0 = 0.0752$ Gyr^{-1}. All these values correspond to the measured values at the current time ($t = 13.6$ Gyr). Thus, with the pass of time both the cosmological constant and its derivative, and with them the energy density and pressure too, will tend to zero. One can easily see that $H \to 0$ for $t -> \infty$ and, hence, those are pseudo-Rip models.





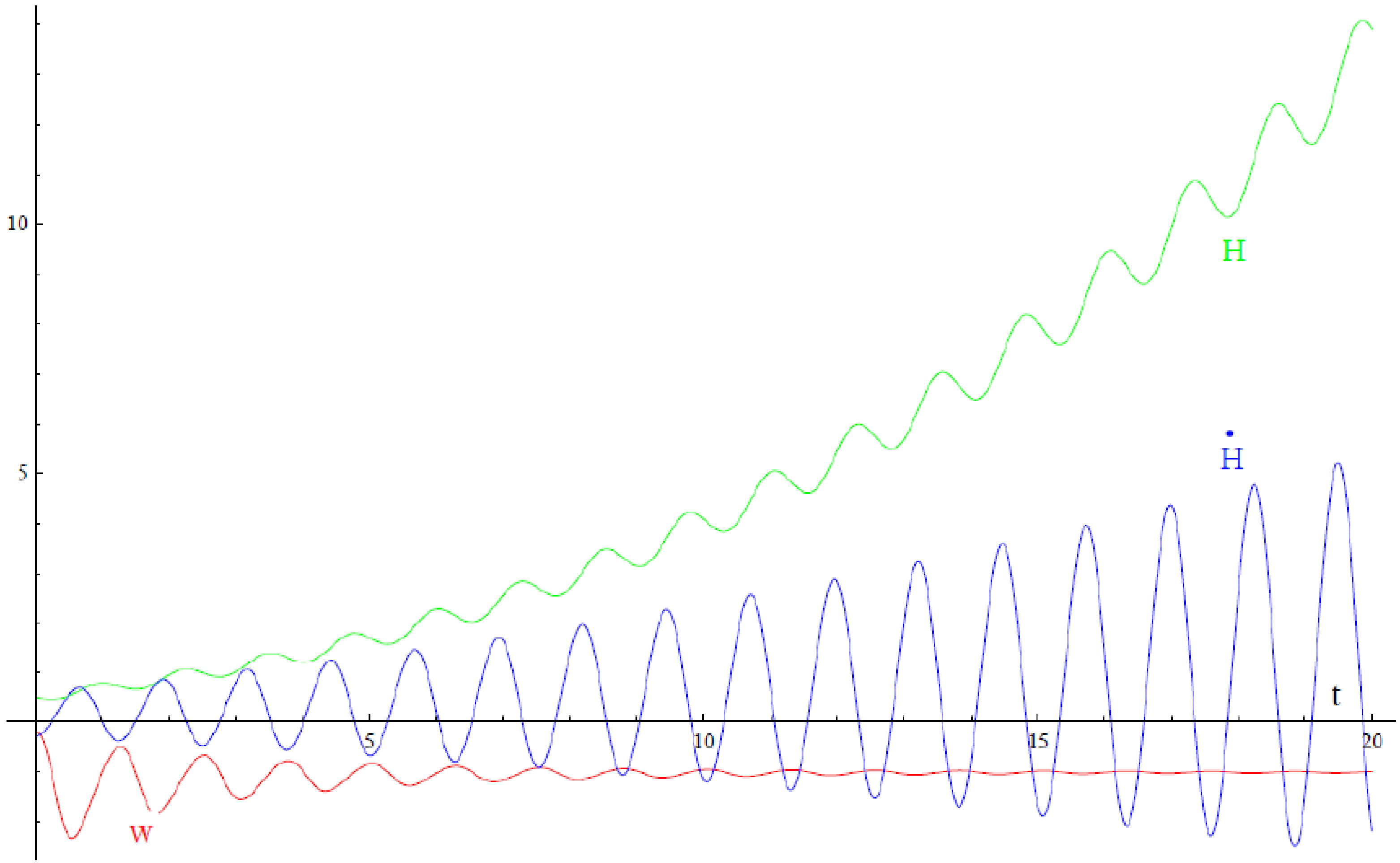
500
400
300
200
100
a
t Gyr
20
40
60
80
100

By selecting different values of the constants one can obtain different behaviors for the EoS Parameter (c_1 =1, c = 0.1, b = 3, q = 3, and $-5 < g < 5$).

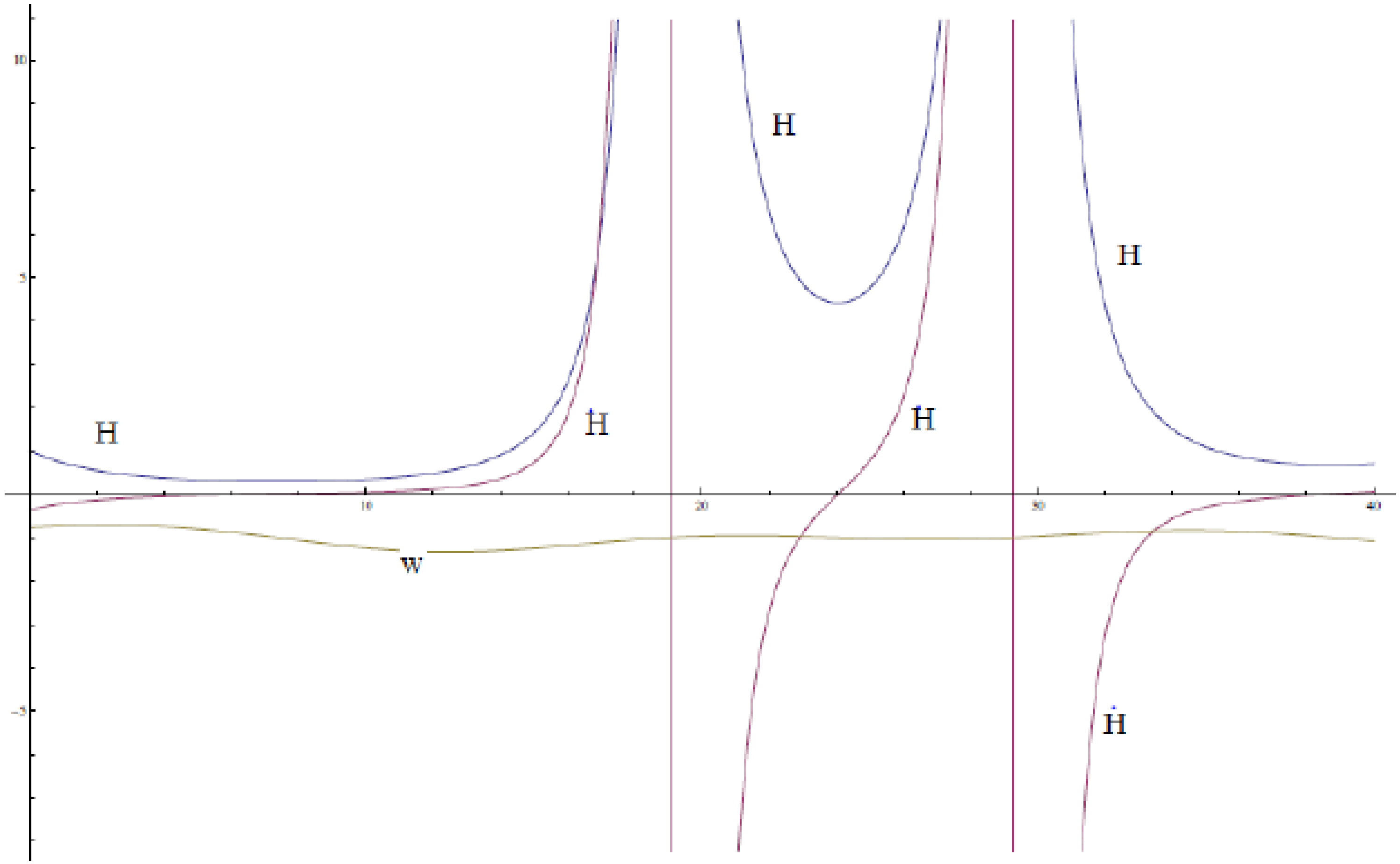


For the earlier values of time one gets accelerated expansion, then the expansion slows down, and later the acceleration will start again.

The oscillation w can acquire values around minus one. This case corresponds to the Little Rip model ($H \to \infty$ for $t \to \infty$) (for $c_1 = 0.1$, $c = 5$, $b = 0.6$, $q = 0.5$, and $g = -3$).



If γ is positive and the parameter c_1 is negative, then we get a singularity in the future.



CONCLUSION

It can be seen that a model of this kind leads to different types of evolution of the universe. First, one can build a model that will consistently describe all the stages in the universe evolution: accelerated expansion, slowing down to $w = 1/3$, and accelerated expansion again, while for $t \to \infty$ the Hubble constant and its derivative tend to zero. Second, one can adjust for the right behavior of the model in the far future: The universe turns to be de Sitter or exhibits one of the four types of singularities. Moreover, almost always is it possible to choose the parameters so that they match the observed values. This is not difficult to do by assuming that at present the universe is in a phase corresponding to an effective cosmological constant. In addition, these models exhibiting multiple cosmological constants, which definitely shows an analogy with the cosmological landscape picture.

Thank you

$$
\begin{aligned}
0 \;&=\; \frac{1}{2k^2}\left(-R^{\mu\nu}+\frac{1}{2}g^{\mu\nu}R\right)+T^{\mu\nu}+\frac{1}{2}g^{\mu\nu}f(G) \\
&-\; 2f_G R R^{\mu\nu}+4f_G R^{\mu}_{\alpha}R^{\nu\alpha}- \\
&-\; 2f_G R^{\mu\alpha\beta\tau}R^{\nu}_{\alpha\beta\tau}-4f_G R^{\mu\alpha\beta\nu}R_{\alpha\beta}+ \\
&+\; 2\left(\nabla^{\mu}\nabla^{\nu}f_G\right)R-2g^{\mu\nu}(\nabla^2 f_G)R- \\
&-\; 4(\nabla_{\rho}\nabla^{\mu}f_G)R^{\nu\rho}-4(\nabla_{\rho}\nabla^{\nu}f_G)R^{\mu\rho}+ \\
&+\; 4(\nabla^2 f_G)R^{\mu\nu}+4g^{\mu\nu}(\nabla_{\rho}\nabla_{\sigma}f_G)R^{\rho\sigma}- \\
&-\; 4(\nabla_{\alpha}\nabla_{\beta}f_G)R^{\mu\alpha\nu\beta},
\end{aligned}
$$