Strong gravity and relativistic accretion disks around supermassive black holes*

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Abstract

Here we used numerical simulations based on ray-tracing method in Kerr metric to study the influence of strong gravity and spin of supermassive black holes on radiation emitted from their relativistic accretion disks. The obtained results showed that space-time metric has a significant influence on the shape of the broad Fe Kα spectral line, emitted from the innermost parts of such relativistic accretion disks. These simulations, when compared with the observed X-ray spectra of active galaxies and quasars, could be used for obtaining the parameters of their central supermassive black holes (e.g. their spins) and for studying the plasma physics, space-time geometry and effects of strong gravity in the central parts of these objects.

1. Supermassive black holes

According to General Relativity, a black hole is a region of space-time around a collapsed mass with a gravitational field that has became so strong that nothing (including electromagnetic radiation) can escape from its attraction, after crossing its event horizon [1]. Black holes have only three measurable parameters, not including the Hawking temperature [2]: charge, mass and angular momentum (spin). Therefore, they can be classified according to the metrics which describe the space-time geometry around them (i.e. according to the corresponding solutions of the field equations of General Relativity) into the following four types:

(i) Schwarzschild (non-rotating and uncharged)
(ii) Kerr (rotating and uncharged)
(iii) Reissner-Nordström (non-rotating and charged)
(iv) Kerr-Newman (rotating and charged)

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Besides, all the black holes in nature are commonly classified according to their masses as (see e.g. [3]):

(i) Mini, micro or quantum mechanical: $M_{BH} \ll M_\odot$ (primordial black holes in the early Universe)

(ii) stellar-mass: $M_{BH} < 10^2\ M_\odot$ (located in the X-ray binary systems)

(iii) intermediate-mass: $M_{BH} \sim 10^2 - 10^5\ M_\odot$ (located in the centers of globular clusters)

(iv) supermassive: $M_{BH} \sim 10^5 - 10^{10}\ M_\odot$ (located in the centers of all galaxies, including ours),

where $M_\odot$ denotes solar mass.

2. Relativistic accretion disks

It is now widely accepted that most galaxies harbor a supermassive black hole (SMBH) in their centers and that the formation and evolution of galaxies is fundamentally influenced by properties of their central SMBHs. Especially interesting are active galaxies which most likely represent one phase in evolution of any galaxy. They derive their extraordinary luminosities from energy release by matter accreting towards, and falling into, a central SMBH through a relativistic accretion disk which represents an efficient mechanism for extracting gravitational potential energy and converting it into radiation. In the case of such SMBHs, only their masses and spins are of sufficient importance because they are responsible for several effects which can be observationally detected (see e.g. [4]). Therefore, the space-time geometry around SMBHs is usually described by Schwarzschild or Kerr metrics.

The innermost regions of a relativistic accretion disk radiate in the X-ray spectral band and the most prominent X-ray feature is a broad emission, double peaked Fe Kα spectral line at 6.4 keV which has an asymmetric shape with narrow bright blue and wide faint red peak [5, 6]. Since this line is produced close to the marginally stable orbit, it is an important indicator of accreting flows around SMBHs, as well as of space-time geometry in these regions (for more details, see [3] and references therein). In the case of some active galaxies, its Full-Widths at Half-Maximum (FWHM) corresponds to relativistic velocities which sometimes reach 30% of the speed of light. Therefore, its complex profile is a result of combination of three different effects (see e.g. [7]):

(i) Doppler shift due to rotation of emitting material, which is responsible for occurrence of two peaks;

(ii) special relativistic effect, known as relativistic beaming, which is responsible for enhancement of blue peak with respect to the red one;

(iii) general relativistic effect, known as gravitational redshift, which is responsible for smearing of the line profile.

3. Ray-tracing method in the Kerr metric

In order to study the effects of strong gravity in the vicinity of SMBHs we developed a code to perform numerical simulations of radiation from relativistic accretion disks around them, based on ray-tracing method in the Kerr metric. In this method we used the pseudo-analytical integration of the geodesic equations which describe the photon trajectories in the general case of a rotating black hole having some angular momentum J (see [8, 9] for more details). In such case space-time geometry is described by the Kerr metric which depends on the spin a of the black hole, i.e. on its angular momentum J normalized to its mass M ($a = J/Mc$, $0 \leq a \leq M$):

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4Mar}{\Sigma}\sin^2\theta dt d\phi + \frac{A}{\Sigma}\sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2, \tag{1}$$

where

$$\Sigma = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 + a^2 - 2Mr, \quad A = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta. \tag{2}$$

Above definition is given in Boyer-Lindquist coordinates for $c = G = 1$. Here $a \leq M$ because any additional angular momentum would increase the energy of the black hole and, therefore, its mass. In the case of a non-rotating black hole, i.e. for $a \to 0$, Kerr metric reduces to the Schwarzschild one.

By solving the equation $\Delta = 0$ and taking only a root with + sign, we obtain the radius of event horizon of a black hole: $r_h = M + \sqrt{M^2 - a^2}$. Thus, when $a \to 0$ then $r_h \to 2M = R_S$ (R_S being Schwarzschild radius), whereas when $a \to M$ (i.e. for a maximally rotating or extreme Kerr black hole) then $r_h \to M$.

The minimum allowed radius r_{ms} of a stable circular equatorial orbit, or so called marginally stable orbit, is given by the roots of the following equation [8]: $r^2 - 6Mr \mp 8a\sqrt{Mr} - 3a^2 = 0$, where the upper sign refers to co-rotating orbits, while the lower one to counter-rotating orbits. In the case of Schwarzschild metric ($a = 0$) $r_{ms} = 6M$, whereas for a maximally rotating black hole ($a = M$) $r_{ms} = M$ [8].

A photon trajectory in the Kerr metric can be described by three constants of motion (the energy at infinity and two constants related to the angular momentum, respectively) which have the following forms when natural units $c = G = M = 1$ are assumed [9]: $E = -p_t$, $\Lambda = p_\phi$ and $Q = p_\theta^2 - a^2E^2cos^2\theta + \Lambda^2cot^2\theta$. Here, (r, θ, ϕ, t) are the usual Boyer-Lindquist coordinates and p is the 4-momentum. As the trajectory of a photon is independent on its energy, it may be expressed using the two dimensionless parameters $\lambda = \Lambda/E$ and $q = Q^{1/2}/E$ which are very simply related to the two impact parameters α and β describing the apparent position on the observer's celestial sphere: $\alpha = -\frac{\lambda}{\sin\theta_{obs}}$ and $\beta = \pm\left(q^2 + a^2\cos^2\theta_{obs} - \lambda^2\cot^2\theta_{obs}\right)^{\frac{1}{2}}$, where the sign of β is determined by $\left(\frac{dr}{d\theta}\right)_{obs}$.

In order to find the photon trajectories (null geodesics) which originate in the accretion disk at some emission radius r_{em} and reach the observer at infinity, one must solve the following integral equation [9]:

$$\pm \int_{r_{em}}^{\infty} \frac{dr}{\sqrt{R\,(r,\lambda,q)}} = \pm \int_{\theta_{em}}^{\theta_{obs}} \frac{d\theta}{\sqrt{\Theta\,(\theta,\lambda,q)}}, \tag{3}$$

where

$$\begin{aligned} R\,(r,\lambda,q) &= \left(r^2 + a^2 - a\lambda\right)^2 - \Delta\Big[(\lambda - a)^2 + q^2\Big], \\ \Theta\,(\theta,\lambda,q) &= q^2 + a^2\cos^2\theta - \lambda^2\cot^2\theta. \end{aligned} \tag{4}$$

The above integral equation (3) can be solved in terms of Jacobian elliptic functions, i.e. by a pseudo-analytical integration. For the exact expressions of the solutions, see e.g. [9].

Due to relativistic effects, photons emitted at frequency ν_{em} will reach infinity at frequency ν_{obs}, and their ratio determines the shift due to these effects: $g = \frac{\nu_{obs}}{\nu_{em}}$. The total observed flux at the observed energy E_{obs} is given by:

$$F_{obs}\,(E_{obs}) = \int\limits_{image} \varepsilon\,(r) g^4 \delta\,(E_{obs} - gE_0)\, d\Xi, \tag{5}$$

where $\varepsilon\,(r)$ is the disk emissivity, $d\Xi$ is the solid angle subtended by the disk in the observer's sky and E_0 is the rest energy.

Using such ray-tracing method, one takes into account only those photon trajectories which reach the observer's sky plane. Computationally, this is much more efficient than the direct integration of the geodesic equations. In the ray-tracing, one divides the image of the disk on the observer's sky into a number of small elements (pixels), and for each pixel the photon trajectory is traced backward from the observer by following the geodesics in a Kerr space-time, until it crosses the plane of the disk. Then, the flux density of the radiation emitted by the disk at that point, as well as the redshift factor of the photon are calculated. In that way, one can obtain the color images of the accretion disk which a distant observer would see by a powerful high resolution telescope (see the left panels of Figs. 1 and 2). The simulated line profiles emitted from such a relativistic accretion disk, can be calculated taking into account the intensities and received photon energies of all pixels over the disk image (see the right panels of Figs. 1 and 2).

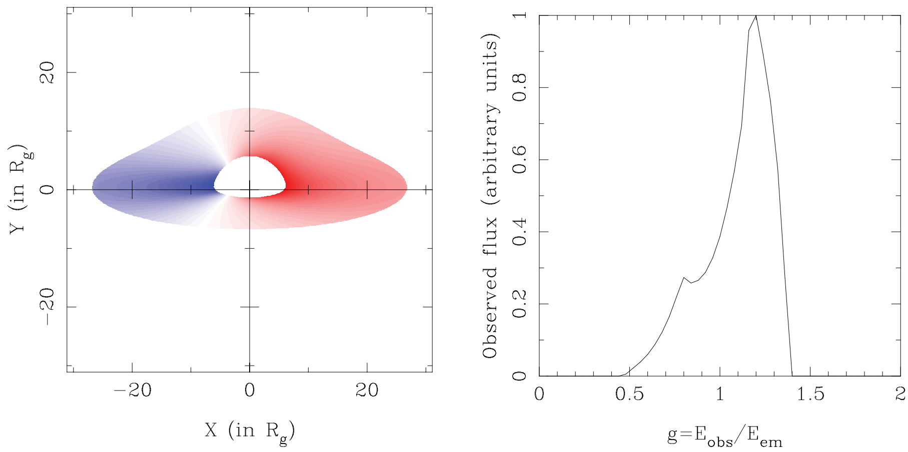

Figure 1: *Left:* a simulated image of a highly inclined ($i = 75°$) accretion disk around Schwarzschild SMBH. The disk extends from the marginally stable orbit around SMBH up to the 30 R_g, where gravitational radius R_g is equal to one half of Schwarzschild radius. *Right:* a simulated profile of the Fe Kα spectral line emitted from the accretion disks presented in the left panel [7].

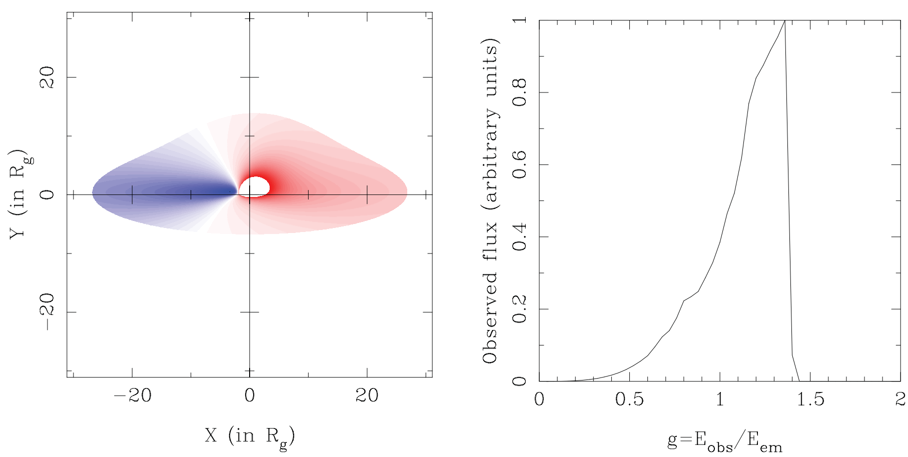

Figure 2: The same as in Fig. 1, but for an extreme (i.e. maximally rotating) Kerr SMBH [7].

4. Observable effects of SMBH spin and strong gravity

The complex profile of the Fe Kα line depends on different parameters of SMBH and its accretion disk. Left panels of Figs. 1 and 2 present the simulated images of a highly inclined ($i = 75°$) accretion disk around a Schwarzschild and an extreme (i.e. maximally rotating) Kerr SMBH. Right panels of these two figures present the corresponding simulated profiles of the Fe Kα spectral line originating from the accretion disks presented in the left panels. As one can see from Figs. 1 and 2, radius of the innermost circular orbit (so called marginally stable orbit) strongly depends on SMBH spin, and consequently, affects the radiation originating from the innermost regions of the disk. The red wing of the line is especially affected, and it extends towards lower energies for higher values of the spin [3].

In order to study the effects of strong gravity on the Fe Kα line, we assumed that the line was emitted from several regions in form of narrow rings located at different distances from SMBH. These emitting regions are presented in the left panel of Fig. 3, and the corresponding line profiles in the right panel of the same figure. From Fig. 3 one can see how the Fe Kα line profile changes as the function of distance from central SMBH. When the line emitters are located at the lower radii of the disk, i.e. closer to the central SMBH, the line is broader due to faster rotation of emitting material, and more asymmetric due to stronger relativistic beaming. When the line emission originates at larger distances from the SMBH, emitting material rotates slower and relativistic beaming is smaller, and therefore the line becomes narrower and more symmetric (see also [4]).

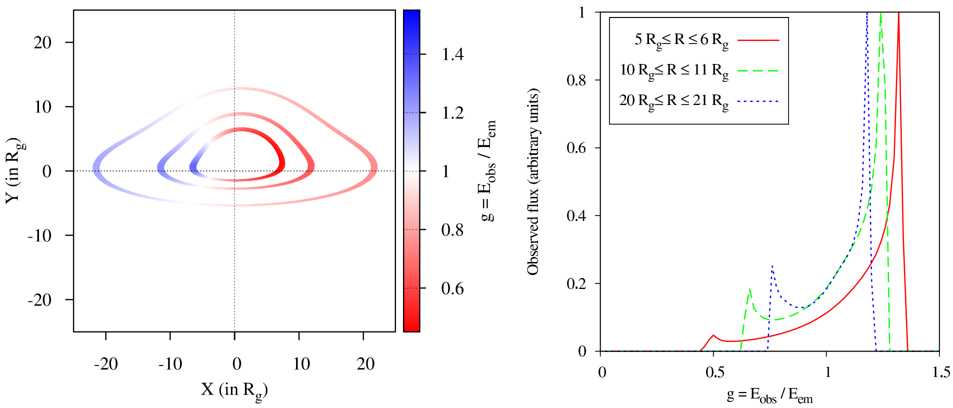

Figure 3: *Left:* The Fe Kα line emitting regions in form of narrow rings extending from $5 - 6$ R_g, $10 - 11$ R_g and $50 - 51$ R_g, located in a highly inclined ($i = 75°$) accretion disk around an extreme Kerr SMBH. *Right:* the corresponding profiles of the Fe Kα line emitted from the narrow rings presented in the left.

5. Simulations versus observations

Since the space-time metric significantly affects the shape of the Fe Kα line, this effect could be used for estimation of SMBH spin by comparisons between the simulated and observed X-ray spectra of active galaxies and quasars. Also, parameters of accretion disks around these SMBHs (such as their size, inclination, emissivity, accretion rate, etc) could be also studied in order to reveal the physics and structure in the central parts of these objects, as well as to get information about their activity, evolution and observational signatures [7].

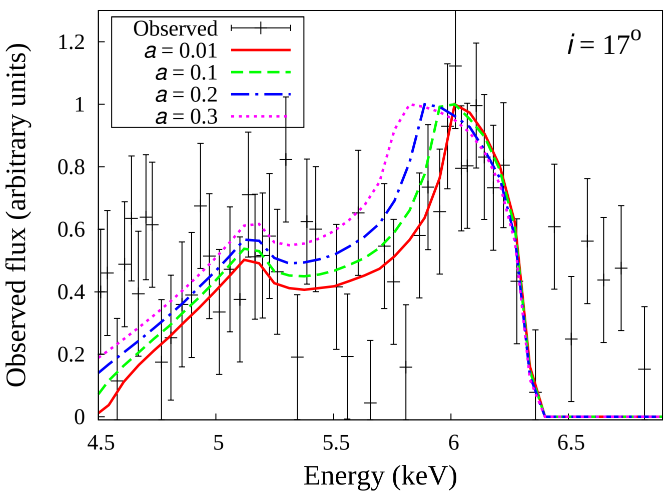

Figure 4: Comparison between the observed and simulated profiles of the Fe Kα line in the case of 3C 405 [10]. Chandra observations [11] are denoted with black crosses with error bars, while the corresponding simulated profiles for 4 different values of SMBH spin are denoted with color lines.

An example for such a comparison between the simulated and observed profiles of the Fe Kα spectral line is given in Fig. 4 (see [10] for more details), in the case of a radio-loud active galaxy 3C 405 (Cygnus A). The observed profiles were obtained by Chandra X-ray observatory space telescope [11]. As it can be seen from Fig. 4, the best fit was obtained for a very slowly rotating Kerr or even a Schwarzschild SMBH and for a slightly inclined accretion disk ($i = 17°$), extending from the radius of the marginally stable orbit up to 10 $R_{\rm g}$.

6. Conclusions

We studied the innermost regions of relativistic accretion disks around central SMBHs of active galaxies and quasars using numerical simulations based on ray-tracing method in Kerr metric. The results of our investigations can be summarized as follows:

(i) numerical simulations based on ray-tracing method in Kerr metric represent a powerful tool for studying the plasma physics, space-time geometry, as well as the effects of strong gravity in the vicinity of SMBHs;

(ii) spin of SMBHs, as a property of the space-time metric, has a significant influence on radiation emitted from the innermost regions of the relativistic accretion disks;

(iii) relativistic beaming also strongly affects this radiation and is responsible for asymmetry of the emitted Fe Kα line;

(iv) comparisons between the simulated and observed shapes of the Fe Kα spectral line enable determination of the parameters of the SMBHs, as well as of their relativistic accretion disks.

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