Orbital precession in R^n gravity: simulations vs observations (the S2 star orbit case)*

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Abstract

In this paper we study some possible observational signatures of R^n gravity at Galactic scales. We make comparison between the theoretical results and observations. For that purpose, we performed computer simulations in R^n gravity potential (modifications of the Newton's gravity law) and analyzed the obtained trajectories of S2 star around Galactic center. Our results show that the most probable value for the parameter r_c in R^n gravity potential in the case of S2 star is $\sim$100 AU, while the universal parameter β is close to 0.01. Also, our results show that the R^n gravity potential induces the precession of S2 star orbit in opposite

* Work supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia through the project 176003 "Gravitation and the large scale structure of the Universe".

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direction with respect to General Relativity. It has a similar effect like extended mass distribution which produces a retrograde shift, that results in rosette shaped orbits.

PACS: 04.50.Kd, 04.80.Cc, 04.25.Nx, 04.50.-h

1. Introduction

Power-law fourth-order theories of gravity have been proposed like alternative approaches to Newtonian gravity [1, 2]. In this work we study possible application of R^n gravity [3] on Galactic scales, for explaining observed precession of orbits of S2 star and make comparison between the theoretical results and observations.

S2 star is one of the S-stars which are the bright stars which move around the massive black hole in the center of our Galaxy [4, 5, 6]. Progress in monitoring bright stars near the Galactic Center have been made recently [7, 4]. The accuracy is constantly improving from around 10 mas during the first part of the observational period, currently reaching less than 1 mas. With that limit one can not say for sure that S2 star orbit really deviates from the Newtonian case.

Capozziello et al. [2] found a very good agreement between the theoretical rotation curves and the observation data using only stellar disc and interstellar gas when $\beta = 0.817$, obtained by fitting the Type Ia supernova Hubble diagram with the assumed power-law $f(R)$ model and without dark matter [2]. Frigerio Martins and Salucci [8] have also investigated the possibility of fitting the rotation curves of spiral galaxies with the power-law fourth-order theory of gravity, without the need for dark matter.

Zakharov et al. [9] found that these parameters must be very close to those corresponding to the Newtonian limit of the theory. In paper [10] the authors discuss the constraints that can be obtained from the orbit analysis of stars (as S2 and S16) moving inside the DM concentration. Rubilar and Eckart [11] showed that the orbital precession can occur due to relativistic effects, resulting in a prograde shift, and due to a possible extended mass distribution, producing a retrograde shift. Weinberg et al. [12] demonstrated that the lowest order relativistic effects, such as the prograde precession, will be detectable if the astrometric precision become less than 0.5 mas.

Talmadge et al. [13] calculated perihelion precession in a potential which deviates from Newtonian potential only slightly. Adkins and McDonnell [14] calculated the precession of Keplerian orbits under the influence of arbitrary central force perturbations. Schmidt [15] calculated the perihelion precession of nearly circular orbits in a central potential.

2. Theory

R^n gravity belongs to power-law fourth-order theories of gravity obtained by replacing the scalar curvature R with $f(R) = f_0 R^n$ in the gravity Lagrangian [1, 2]. As a result, in the weak field limit [16], the gravitational potential is found to be [1, 2]:

$$\Phi\left(r\right) = -\frac{GM}{2r}\left[1 + \left(\frac{r}{r_c}\right)^{\beta}\right], \tag{1}$$

where r_c is an arbitrary parameter, depending on the typical scale of the considered system and β is a universal parameter.

This corresponds to a modification of the gravity action in the form:

$$A = \int d^4x\sqrt{-g}\left(f\left(R\right) + L_m\right), \tag{2}$$

where $f(R)$ is a generic function of the Ricci scalar curvature and L_m is the standard matter Lagrangian.

Parameter β controls the shape of the correction term. Since it is the same for all gravitating systems, as a consequence, β must be the same for all of them and therefore it is universal parameter [2]. The parameter r_c is the scalelength parameter and is related to the boundary conditions and the mass of the system [2].

3. Results and discussions

In the Fig. 1 (left and right) are presented the orbits of S2-like star around massive black hole in R^n gravity (blue solid line) and in Newtonian gravity (red dashed line) for $r_c = 100$ AU and $\beta = 0.01$ during 0.8 and 10 periods, respectively. The precession of S2 star orbit is in the clockwise direction in the case when the revolution of S2 star is in counter clockwise direction.

We assume that R^n gravity potential does not differ significantly from Newtonian potential and we will derive formula for precession angle of the modified orbit, during one orbital period. First step is to derive perturbing potential from:

$$V(r) = \Phi\left(r\right) - \Phi_N\left(r\right)\ \ ;\ \ \Phi_N\left(r\right) = -\frac{GM}{r}\ . \tag{3}$$

Obtained perturbing potential is of the form:

$$V(r) = -\frac{GM}{2r}\left(\left(\frac{r}{r_c}\right)^{\beta} - 1\right), \tag{4}$$

and it can be used for calculating the precession angle according to the equation (30) from paper [14]:

$$\Delta\theta = \frac{-2L}{GMe^2}\int\limits_{-1}^{1}\frac{z\cdot dz}{\sqrt{1-z^2}}\frac{dV\left(z\right)}{dz}, \tag{5}$$

where r is related to z via: $r = \dfrac{L}{1+ez}$. By differentiating the perturbing potential $V(z)$ and substituting its derivative and expression for the semilatus rectum of the orbital ellipse ($L = a\left(1-e^2\right)$) in above equation (5), we obtain:

$$\Delta\theta = \frac{\pi}{2}\beta\left(\beta-1\right)\left(\frac{a\left(1-e^2\right)}{r_c}\right)^{\beta} \times {}_2F_1\left(\frac{\beta+1}{2}, \frac{\beta+2}{2}; 2; e^2\right), \qquad (6)$$

where ${}_2F_1$ is hypergeometric function.

Exact expression (6) is inappropriate for practical applications. However, it can be easily approximated for $\beta \approx 0$. In case of $\beta \approx 0$ expansion of Eq. (6) in Taylor's series over β, up to the first order, leads to the following expression for precession angle:

$$\Delta\theta = \frac{180^\circ\beta\left(\sqrt{1-e^2}-1\right)}{e^2}. \qquad (7)$$

As it can be seen from formula (7), the precession angle in the case when β is small ($\beta \approx 0$) depends only on eccentricity and universal constant β itself.

Comparison of the precession angles obtained analytically from approximative formula (7) and obtained numerically from calculatedorbits is presented in Table I in paper [17]. As it can be seen from that table, the approximative formula (7) can be used for estimating the precession angle when β is small (values of β of interest, see ref. [17]).

Here we compare the obtained theoretical results for S2 star orbits in the R^n potential with two independent sets of observations of the S2 star, obtained by New Technology Telescope/Very Large Telescope (NTT/VLT), as well as by Keck telescope which are publicly available as the supplementary on-line data to the electronic version of the paper [4]. However, all the above simulations in R^n gravity potential resulted with the true orbits of S2 star. In order to compare them with observed positions, the first step is to project them to the observer's sky plane, i.e. to calculate the corresponding apparent orbits [17].

The best fit is obtained for the following small value of the universal constant: $\beta = 0.01$, in which case the corresponding precession is around -1°. In Fig. 2 we present two comparisons between the obtained best fit orbit for $\beta = 0.01$ in the R^n gravity potential and the positions of S2 star observed by NTT/VLT (left) and Keck (right) in vicinity of its apocenter. We achieved the satisfying agreement between the same fitted orbit and the both NTT/VLT and Keck data sets.

As it can be seen from Fig. 2, the orbit of S2 is not closed in vicinity of its apocenter, which clearly shows that the orbital precession is a natural consequence of R^n gravity. Moreover, by comparing the arcs of orbit near the apocenter with the corresponding results presented in Fig. 1 from [4], one can see that their curvatures are different, which indicates the opposite directions of precession in these two cases.

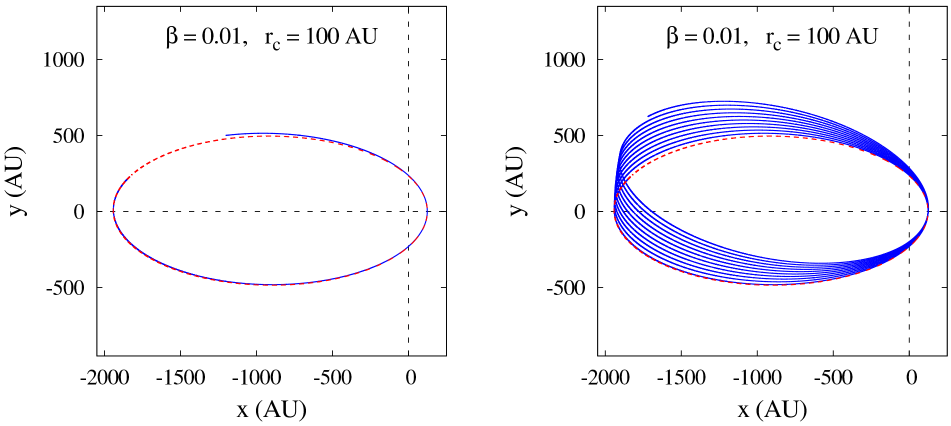

Figure 1: The orbits of S2-like star around massive black hole in R^n gravity (blue solid line) and in Newtonian gravity (red dashed line) for $r_c = 100$ AU and $\beta = 0.01$ during 0.8 periods (*left*) and 10 periods (*right*).

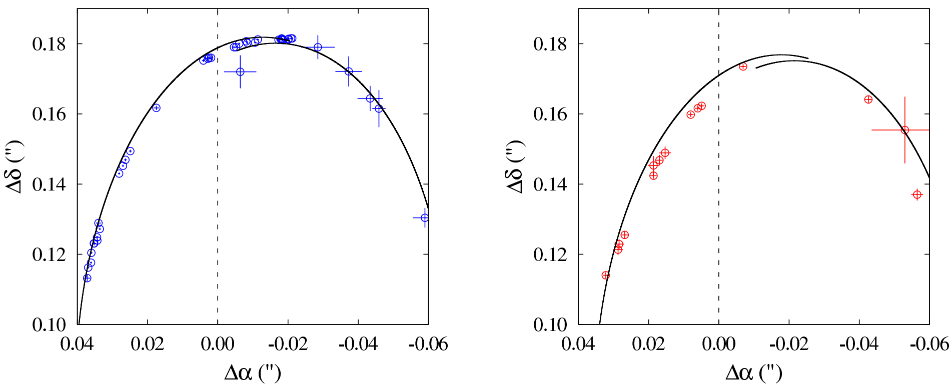

Figure 2: The fitted orbit of S2 star around massive black hole in R^n gravity for $r_c = 100$ AU and $\beta = 0.01$ (black solid lines in both panels) in vicinity of its apocenter. The NTT/VLT astrometric observations are presented in the left panel by blue circles, while the Keck measurements are denoted by red circles in the right panel.

4. Conclusions

In this paper we calculated orbits of S2 star in the R^n gravitation potential. Using the observed positions of S2 star we determined the parameters of this class of gravity theories.

Despite the excellent agreement between theoretical and observed rotation curves obtained by Capozziello and coworkers [2] for R^n parameter $\beta = 0.817$, our findings indicate that for $\varepsilon = 0''.01$ maximal value of β is 0.0475, and our fitting indicated that optimal value for β is around 0.01 [17]. Therefore, R^n gravity in this form may not represent a good candidate to solve both the dark energy problem on cosmological scales and the dark matter one on galactic scales using the same value of parameter β. But this theory has its own benefits in explaining orbits of the stars and solar system data.

The newest astrometric data for the star S2 of NTT/VLT measurements and Keck measurements indicate that maybe the S2 orbits do not yield closed ellipses. We compared these data with S2 star orbit obtained using R^n gravity potential. We can conclude that additional term in R^n gravity compared to Newtonian gravity has a similar effect like extended mass distribution and produce a retrograde shift, that results in rosette shaped orbits. We hope that future observations with advanced facilities, such as GRAVITY[18] or METIS[19], will be able to verify these claims.

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